Moldova State University
Faculty of Mathematics and Computer Science
Mathematical Society of the Republic of Moldova

INTERNATIONAL CONFERENCE

MATHEMATICS & INFORMATION TECHNOLOGIES: RESEARCH AND EDUCATION (MITRE-2023)

Satellite conference
of the Tenth Congress of Romanian Mathematicians

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This book includes the abstracts of communications, presented at the Conference "Mathematics & Information Technologies: Research and Education", 9th edition, held at the Moldova State University, Chişinău, on June 26-29, 2023.

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INTERNATIONAL CONFERENCE MITRE-2023

The International Conference "*Mathematics & IT: Research and Education (MITRE-2023)*" is organized by the Faculty of Mathematics and Computer Science, the Center for Education and Research in Mathematics and Computer Science (CECMI) of the Moldova State University.

The Conference MITRE in 2023 is at its 9^{th} edition.

The main goal of the Conferences MITRE is to provide a forum for specialists to discuss different aspects of the integration of research and education in Mathematics and Computer Science. The discussion is planed to be axed on the achievements in scientific research and advanced training of high qualification specialists in these areas, according to real needs of economy, on ways of involving young talents in research.

The Conference is organized in five sections:

1. Algebra, Geometry and Topology
2. Analysis, Differential Equations and Dynamical Systems
3. Applied Mathematics
4. Computer Science and IT
5. Didactics of Mathematics and Informatics

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This book contains the abstracts of communications, presented at the Conference MITRE-2023 (some communications in the Section Didactics of Mathematics and Informatics are presented in Romanian). The authors are responsible for the content of their abstract. The authors are listed in the alphabetical order by the last name of the first author within the conference sections.

The Organizing Committee MITRE-2023 thanks the authors for contribution with their abstracts.

PLENARY LECTURES

BIAUTOMATICITY OF HELLY AND CB-GROUPS

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In the talk, we will outline the proofs that Helly groups and CB-groups are biautomatic. Helly groups are the groups acting geometrically on Helly graphs and CB-groups are the groups acting geometrically on graphs with convex balls. Helly groups represent a common generalization of hyperbolic groups, CAT(0) cubical groups, graphical C(4)-T(4) small cancellation groups, swm-groups, and some other classes of groups. CB-groups generalize weakly systolic and systolic groups. Both proofs of biautomiaticity use a result of Świątkowski (2006) providing sufficient conditions of biautomaticity in terms of local recognition and bicombing and a construction of normal clique paths in the graph on which the group acts. The normal clique paths in Helly graphs and in CB-graphs are defined using different properties of such graphs. In case of CB-graphs, our construction generalizes the construction of Januszkiewicz and Świątkowski (2006) for systolic groups. The talk is based on the following papers:

References:

1. J. Chalopin, V. Chepoi, A. Genevois, H. Hirai, and Damian Osajda. Helly groups. *Geometry and Topology* (to appear), arXiv preprint arXiv:2002.06895 (2020).
2. J. Chalopin, V. Chepoi, and U. Giocanti. Graphs with convex balls. *Geometriae Dedicata* (to appear), arXiv preprint arXiv:2201.01599 (2022).

LYAPUNOV EXPONENTS, ANOSOV REPRESENTATIONS, AND HODGE THEORY

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Discrete subgroups of semisimple Lie groups arise in a variety of contexts, sometimes “in nature” as monodromy groups of families of algebraic manifolds, and other times in relation to geometric structures and associated dynamical systems. I will explain a method to establish that monodromy groups of certain variations of Hodge structure give Anosov representations, thus relating algebraic and dynamical situations. Among many consequences of these interactions, I will explain a proof of a conjecture of Eskin, Kontsevich, Moller, and Zorich on Lyapunov exponents, some uniformization results for domains of discontinuity of the associated discrete groups, and Torelli theorems for certain families of Calabi-Yau manifolds (including the mirror quintic). The discrete groups of interest live inside the real linear symplectic group, and the domains of discontinuity are inside Lagrangian Grassmanians and other associated flag manifolds. The necessary context and background will be explained.

A PERRON-FROBENIUS THEOREM FOR PRODUCTS OF NON NEGATIVE RANDOM MATRICES WITH APPLICATIONS

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Let $d \geq 1$ and $\mathbb{R}^d$ be the d-dimensional vector space endowed with orthonormal basis $(e_i)_{1\leq i\leq d}$ and with the scalar product $\langle\cdot,\cdot\rangle$ and the L^1 norm $\|\cdot\|$. We denote by $\mathcal{G}$ the multiplicative semigroup of $d\times d$ matrices with real non-negative entries which are allowable (that is every row and every column has a strictly positive entry). Let $(\Omega,\mathcal{F},\mathbb{P})$ be a probability space and let $(A_k)_{k\in\mathbb{Z}}$ be a sequence of stationary and ergodic random elements of $\mathcal{G}$ of the same law indexed in $\mathbb{Z}$. The object of study is the left product matrix $A_n\cdots A_k$, for $k,n\in\mathbb{Z}$ and $k\leq n$. Our goal is to establish an analog of the Perron-Frobenius theorem for products of random matrices.

We assume that the sequence of matrices $(A_n)_{n\in\mathbb{Z}}$ in $\mathcal{G}$ satisfies following condition due to Hennion (1997): $\mathbb{P}(\exists n\geq 1, A_n\ldots A_1>0)>0$. Consider the σ-algebras: $\mathcal{F}_k=\sigma\{A_k,A_{k-1},\ldots\}$ and $\mathcal{F}^k=\sigma\{A_k,A_{k+1},\ldots\}$, $k\in\mathbb{Z}$.

Theorem. *Assume the Hennion condition. Then:*
1. There exists a sequence $(u_k)_{k\in\mathbb{Z}}$ such that, the random variable u_k is $\mathcal{F}_k$-measurable, satisfies $u_k>0$ and $A_k^T\cdot u_{k+1}=u_k$, $k\in\mathbb{Z}$.
2. There exists a sequence $(v_k)_{k\in\mathbb{Z}}$ such that, the random variable v_k is $\mathcal{F}^k$-measurable and satisfies $v_k>0$ and $A_k\cdot v_{k+1}=v_k$, $k\in\mathbb{Z}$.
3. For any $k\in\mathbb{Z}$, as $n\to\infty$ and, for any $n\in\mathbb{Z}$, as $k\to-\infty$,

$$\sup_{x,y\in\mathcal{S}}\left|\frac{\langle y,A_n\ldots A_k x\rangle}{\langle\mathbf{1},A_n\ldots A_k\mathbf{1}\rangle\langle u_k,x\rangle\langle v_n,y\rangle}-1\right|\to 0\quad\mathbb{P}\text{-}a.s. \tag{1}$$

Theorem 1 is an extension of the results of Hennion 1997 and Furstenberg and Kesten 1960. We shall discuss an application of this result to multitype branching processes in random environment, which generalizes the famous Kesten-Stigum theorem, see Kesten and Stigum 1966. In particular we improve the recent result in Grama, Liu, Pin 2023 where the construction of the associated martingale is performed.

References:

1. H. Hennion. Limit theorems for products of positive random matrices. *The Annals of Probability*, **25**, 4 (1997), 1545-1587.
2. H. Furstenberg, H. Kesten. Products of random matrices. *The Annals of Mathematical Statistics*, **31,** 2 (1960), 457-469.
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4. H. Kesten H., B.P. Stigum. A Limit Theorem for Multidimensional Galton-Watson Processes. *Ann. Math. Stat.*, **37,** 5 (1966), 1211–1223.

KHOVANOV HOMOLOGY AND FOUR-DIMENSIONAL TOPOLOGY

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Over the last forty years, most progress in four-dimensional topology came from gauge theory and related invariants. Khovanov homology is an invariant of knots in R^3 of a different kind: its construction is combinatorial, and connected to ideas from representation theory. There is hope that it can tell us more about smooth 4-manifolds; for example, Freedman, Gompf, Morrison and Walker suggested a strategy to disprove the $4D$ Poincare conjecture using Rasmussen's invariant from Khovanov homology. It is yet unclear whether their strategy can work. I will explain several recent results in this direction and some of the challenges that appear. A key problem is to certify when a knot is slice (bounds a disk in four-dimensional half-space), which can be tackled with machine learning. The talk is based on joint work with Sergei Gukov, Jim Halverson, Marco Marengon, Lisa Piccirillo, Fabian Ruehle, Mike Willis, and Sucharit Sarkar.

THE VISION OF CLIMATE-NEUTRAL GOVERNANCE IN SMART CITIES

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We live with the concept of Smart Cities for several years, and the main aspects are concentrating on people's quality of life, efficient resource usage, and management in a city, flexible collaboration between different entities, everything being designed to be smart and green. It is estimated that, worldwide, cities are responsible for almost 75% of global resource consumption [1,2].

As a consequence, important challenges and opportunities are considered by the EU for local and regional authorities focusing especially on adapting to climate change.

The main objective of the Climate-Neutral Governance project is to enforce climate-neutral policies and strategies in smart cities, to create a strategy for city governance in Romania given local and regional processes and implementation, validation and refining governance solutions of use-cases offering circular economy introduction of solutions to the Net Zero transition with standards and guidelines for process handling, adoption, and impact monitoring (see Figure 1).

Figure 1: *Integrated and multi-level governance models and policies*

The focus of main objective realization is on *Environment* (energy saving, renewable energy, and reduction of pollution, CO_2 emissions), *Mobility* (accessible and safe transport; integrated mobility system), *Citizenship* (life-long learning and education; nurturing of cultural diversity; civic engagement and citizens'

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creativity), *Living* (safeguarding individual and public health, welfare, cultural and tourist policies, social cohesion), *Government and political participation* (transparent decision-making, accessible online services, political participation), and *Economy* (flexible labor market, entrepreneurship, and innovation).

References:

1. EEA Report 01/2017 – *Climate change, impacts and vulnerability in Europe*, 2016.
2. EEA Report 12/2020 – *Urban adaptation in Europe: how cities and towns respond to climate change*, 2020.
3. European Committee of Regions, *Adapting to climate change: Challenges and opportunities for the EU local and regional authorities*, 2020.

WELL-POSEDNESS CONCEPTS FOR NONLINEAR PROBLEMS

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We start by recalling the concepts of well-posedness for minimizations problems, in the sense of Tykhonov [1] and Levitin-Polyak [2]. We proceed with well-posedness concepts for variational and hemivariational inequalities. These concepts are based on two main ingredients: the existence of a unique solution and the convergence to it of the so-called approximating sequence.

Inspired by these properties, we define a new mathematical object, the so-called Tykhonov triple, denoted by $\mathcal{T}$. Then, we introduce a new concept of well-posedness for abstract problems in metric spaces, the so-called $\mathcal{T}$-well-posedness concept. This concept extends the classical well-posedness concepts for minimization and variational inequalities problems, which represent particular cases of $\mathcal{T}$-well-posedness, associated to a particular Tykhonov triple. The theory of $\mathcal{T}$-well-posedness problems we construct gives necessary and sufficient conditions which guarantee the convergence to the solution of a nonlinear problem, unifies different convergence results and provides a framework in which the link between different problems can been established. It can be used in the study of a large class of nonlinear problems like fixed point problems, minimization problems, inequality problems, inclusions, for instance. Details can be found in our furthcomming book [3].

Finally, we illustrate the theory in the study of a class of variational inequalities in reflexive Banach spaces, for which we present a strategy which allows us to deduce various convergence results. As an application we consider a mathematical model which describes the equilibrium of a spring-rods system. The weak formulation of the model is in a form of an elliptic variational inequality in which the unknown is the displacement field. We apply our results in the study of this contact problem and derive convergence results together with the corresponding mechanical interpretations. We also present numerical simulations which validate these convergence results. In this way we fully illustrate the cross fertilization between the abstract mathematical concepts, on one hand, and their applications in Contact Mechanics, on the other hand.

References:

1. A.N. Tykhonov. On the stability of functional optimization problems. *USSR Comput. Math. Math. Phys.*, **6** (1966), 631–634.
2. E.S. Levitin, B.T. Polyak. Convergence of minimizing sequences in conditional extremum problem. *Soviet Math. Dokl.*, **7** (1966), 764–767.
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SEPARATION PROPERTIES OF CONVEX SETS

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Separation properties of convex sets in finite-dimensional vector spaces play an important role in many fields of theoretical and applied mathematics. In this talk, we will discuss both classical and new results on various types of separation of convex sets, possibly non-closed and unbounded.

OPTIMALITY CONDITIONS AND LAGRANGE MULTIPLIERS FOR SHAPE AND TOPOLOGY OPTIMIZATION PROBLEMS

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We discuss first order optimality conditions for geometric optimization problems with Neumann boundary conditions and boundary observation. The methods we develop here are applicable to large classes of state systems or cost functionals. Our approach is based on the implicit parametrization theorem and the use of Hamiltonian systems. It establishes equivalence with a constrained optimal control problem and uses Lagrange multipliers under a new simple constraint qualification. In this setting, general functional variations are performed, that combine topological and boundary variations in a natural way.

I. ALGEBRA, GEOMETRY AND TOPOLOGY

FINITELY SUPPORTED GROUP-VALUED FUZZY SETS AND GENERALIZED MULTISETS

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Generalized multisets are defined as multisets with possibly negative multiplicities, i.e. as functions from a crisp set to the group $\mathbb{Z}$ of all integers. More generally, group-valued fuzzy sets are defined as functions from a classical set to an arbitrary group. The goal of this presentation is to describe the group-valued fuzzy sets (in particular, the generalized multisets) in the framework of finitely supported structures in order to provide a finitary characterization of group-valued fuzzy sets over infinite universes of discourse.

Finitely supported sets [1,2] are related to the permutation models of Zermelo-Fraenkel set theory with atoms (ZFA). These sets can also be described in Zermelo-Fraenkel (ZF) set theory by equipping ZF sets with actions of a group of permutations of some basic elements called atoms. These sets allow a finitary representation of possibly infinite sets containing enough symmetries to be concisely handled. More specifically, they allow us to treat as equivalent those elements having a certain degree of similarity and to focus only on those elements that are 'really different' (by using the notion of 'finite support').

Generally, mappings from a crisp set to an algebraic structure P are called P-fuzzy sets. Zadeh introduced the concept of fuzzy sets by considering P to be the unit interval [0,1]; his theory was generalized by replacing the unit interval [0,1] with an arbitrary complete lattice, and so defining the L-fuzzy sets. It is a natural next step to study the properties of P-fuzzy sets having set P equipped with some internal algebraic law. Here we focus to the description of G-fuzzy sets with G a group; these sets are analyzed in the framework of finitely supported structures.

We introduce finitely supported groups, present some relevant examples of these structures and prove several isomorphism, embedding and universality theorems. We define the finitely supported free groups, and provide a characterization property for the supports of their elements. Then we introduce and study finitely supported G-fuzzy sets, where G is a finitely supported group. We connect the concepts of G-fuzzy set, free group and generalized multiset via some properties derived from category theory. Finally, we present several Dedekind-finiteness properties for infinite G-fuzzy sets.

References:

1. A. Alexandru and G. Ciobanu. *Finitely Supported Mathematics: An Introduction.* Springer, 2016.
2. A. Alexandru and G. Ciobanu. *Foundations of Finitely Supported Structures: A Set Theoretical Viewpoint.* Springer, 2020.

SOME NEW THEOREMS IN SPHERICAL GEOMETRY

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We discuss some new results in spherical geometry. We use theorems from [1-2] in the proof of the following theorems.

Theorem 1. *On a unit sphere with center O, a great circle and one of its poles B is drawn. Two great circles through B intersect the first great circle at Q_1 and Q_2, so that $\widehat{Q_1Q_2} < \frac{\pi}{2}$. Another great circle intersects arcs BQ_1 and BQ_2 at points P_1 and P_2 so that $\widehat{P_1P_2} = \widehat{Q_1Q_2}$. This great circle also intersects the first great circle at T. Then*

$$\widehat{P_1T} + \widehat{TQ_2} = \widehat{P_2T} + \widehat{TQ_1} = \frac{\pi}{2}.$$

Theorem 2. *Under conditions of Theorem 1, let a small circle of the unit sphere be circumscribed about $\triangle BP_1P_2$. If radius of the small circle is R, then*

$$\tan^2 R = \frac{1}{4} \sec^4 \frac{1}{2}\widehat{P_1P_2} \sec^4 \frac{1}{2}\widehat{BP_1} \sec^4 \frac{1}{2}\widehat{BP_2}.$$

The results have some surprising connections with Mercator and Stereographic projections, Gudermannian function, logarithmic and spherical spirals.

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1. M.A. Whittlesey. *Spherical Geometry and Its Applications.* Textbooks in Mathematics. CRC Press: Boca Raton, FL, USA, 2020.
2. K. Odani. On a Relation between Sine Formula and Radii of Circumcircles for Spherical Triangles. *Bull. Aichi Univ. Educ. Natural Sci.*, **59** (2010), 1–5.

[2] *Speaking author*: Y.N. Aliyev

ON FACTOR RINGS OF COMPLETE TOPOLOGICAL RINGS

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The question about the completeness of topological groups and topological rings under various constructions is one of interest in the theory of topological groups and topological rings (see, for example, [1], §3.2 and §4.1). Among the results obtained by many authors, we note the following ones:

– *any closed subgroup of a complete topological Abelian group is complete* (see [1], Theorem 3.2.16);

– *let* (G, τ) *be a topological Abelian group and* A *be its subgroup such that* $(A, \tau|_A)$ *and* $(G, \tau)/A$ *be complete groups, then the topological group* (G, τ) *is complete too* (see [1], Proposition 3.2.22);

– *let a topological Abelian group be complete and satisfies the first axiom of countability, then its quotient group by any closed subgroup is a complete group* (see [1], Proposition 3.2.20);

–*any topological Abelian group whose topology is Hausdorff is isomorphic to the factor-group of some complete topological group* (see [1], Theorem 4.1.48).

Remark. Since the completeness of a topological ring is determined by the completeness of its additive group, the results for topological rings are similar to those given above for topological groups. The analogues of the first three statements noted above are true for topological rings, but the validity of the full analogue of the last fourth is still unknown (the main difficulty here is to construct an appropriate complete ring).

An attempt to prove this was made in monograph [1] (see [1], Theorem 4.1.49). Unfortunately, an error was made in proving the result: the mapping p indicated there is not a ring homomorphism. This error was kindly reported to us by Professor Pace Nielsen. Below we present the valid version of the proof of this theorem, for the case when the topological ring (R, τ) is a locally bounded ring.

Theorem. *Any locally bounded topological ring with the Hausdorff topology is isomorphic to the quotient ring of some complete topological ring.*

[3] *Speaking author*: V.I. Arnautov

References:

1. V.I. Arnautov, S.T. Glavatsky, A.V. Mikhalev. *Introduction to the Theory of Topological Rings and Modules.* Marcel Dekker, Inc., New York, NY, United States, 1996.

COMPUTATION OF GENERALIZED INVERSES OF TENSORS OVER COMMUTATIVE RINGS

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One of the basic operations in linear algebra is matrix multiplication; it expresses the product of two matrices to form a new matrix. A tensor is a higher-dimensional generalization of a matrix (i.e., a first-order tensor is a vector, a second-order tensor is a matrix), but there are more than one way to multiply two tensors (see, e.g. [1-2]). In mathematics, the first axiomatic definition of a ring was given in 1914, in Fraenkel's paper "On zero divisors and the decomposition of rings". In Fraenkel's paper, the author founded the modern definition of abstract ring. Hence, a ring is an algebraic structure with two binary operations (addition and multiplication) over a set satisfying certain requirements. This structure facilitates the fundamental physical laws, such as those underlying special relativity and symmetry phenomena in molecular chemistry It is well-known that the tensors are a multi-dimensional array of numbers, sometimes it is called a multi-way or a multi-mode array. For example, a matrix is a second order tensor or a two way tensor. An element $a = (a_1, a_2, ..., a_p)^T$, where the entries $a_1, a_2, ..., a_p$ are elements from the field of real numbers R, is called a first order tensor with entries from R. Following standard notation, the set of all first order tensor with entries from the field of real number R is denoted by $\mathbb{R}^p$. For convenience we use the known notation $\mathbb{R}^p$ for the ring $(\mathbb{R}^p, +, \odot)$. Also, we refer to a closed multiplication operation between tensors as the t-product. The outline of the paper is computing of generalized inverse of tensors via t-product with apply a von Neumann regular property of tensors over a commutative ring.

Theorem 1. *Let $a, b, c \in \mathbb{R}^p$. Then the following statements are equivalent:*
(i) a is von Neumann regular,
(ii) $\mathbb{R}^p = a \odot b \odot \mathbb{R}^p + rann(c) = \mathbb{R}^p \odot c \odot a + lann(b)$,
(iii) $\mathbb{R}^p = a \odot b \odot \mathbb{R}^p \oplus rann(c) = R^p \odot c \odot a \oplus lann(b)$.

Theorem 2. *Let $a, b, c \in \mathbb{R}^p$ and (b, c)-inverse of a. Then a has a von Neumann regular property.*

Theorem 3. *Let $a, b, c \in \mathbb{R}^p$ and (b, c)-inverse of a. Then a von Neumann regular property of a is unique.*

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GEODESICS ON 2-DIMENSIONAL HYPERBOLIC MANIFOLDS

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In this paper we review some important results on the behavior of geodesics on some hyperbolic surfaces and discuss some extension of those results to the case of an arbitrary hyperbolic surface. Special thanks go to Professor V.S. Makarov who inspired me to do this work. The results offer only a first glimpse on the behavior of geodesics on an arbitrary 2-dimensional hyperbolic manifold. A geodesic in a hyperbolic surface is an arc which in the local coordinate charts, is the image of a geodesic arc of the hyperbolic plane. Topological surfaces are often thought of as the result of pasting togethers polygons. Provided you have enough topology, pants decompositions are a natural way of decomposing (orientable) surfaces or conversely one can build a surface by pasting 3 holed spheres (pants) a long their cuffs. So it is important to study the behavior of geodesics on a pair of pants. Thanks to the development of the new constructive approach, in this paper, the author succeeded to receive "in a certain sense" the solution for the behavior of the geodesics in general on the hyperbolic manifolds, structure of geodesics and their types. Arbitrary hyperbolic surfaces M, closed or open, of finite or infinite genus are considered. Yet another way to define a hyperbolic surface is via its universal cover.

For the behavior of the geodesics on the specified fragments (hyperbolic pants, etc.) it is used a certain figure, named in the text of the work *the multilateral.* The study of the behavior of the geodesics in this paper is being carried out gradually, in order of collecting the surface, the reverse order of cutting the surface into fragments (i.e. pants). The surface is cut into typical pieces (for example, on pants or their degenerations, on right hexagons, etc.) and the question of the behavior of the geodesics for each piece is solved on it, and then the result of the investigation returns (by gluing) onto the original surface. With the help of these multilateral, it is possible to determine the nature of the behavior of the geodesics on the surface. Any given hyperbolic (closed, i.e., ordinary) surface can be cut into pants and the question is how, when gluing such pants, connect them on a common surface. But it may seem (when gluing of the surface from the pants is not finished yet) that the surface of genus g has also n components (the surface has a geodesic boundary). And, going further, we notice that the boundary of the surface can degenerate: transform into cuspidal ends (cusps) and into conical points. Thus, we arrive at the most general case, the surfaces of the signature (g, n, k), the preliminary investigation of the behavior of the geodesics on these pieces. A concrete method of investigating the behavior of the geodesics on hyperbolic 2-manifolds is based on the idea of preliminary research on these pieces (on the set of hyperbolic pants and their degenerations), in the subsequent consolidation of research results using the method proposed in this paper (sometimes called the method of generalized colored multilateral). A new constructive method for investigating the global behavior of the geodesics on hyperbolic manifolds (the method of color multilateral) is given in this paper. The solution is based on the study of the behavior of the geodesics on the simplest hyperbolic surfaces, some of which have long attracted the attention of geometers.

COMMUTATIVE MONOID OF REFLECTIVE FUNCTORS

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In the subcategory $\mathcal{C}_2\mathcal{V}$ of topological vector locally convex Hausdorff spaces (see [1]) classes of reflector (coreflector) functors, every two of which commute, are built.

For a class of monomorphisms $\mathcal{M}$, and a class of objects (a subcategory) $\mathcal{A}$, we denote by $\mathbf{S}_{\mathcal{M}}(\mathcal{A})$ the full subcategory consisting of all $\mathcal{M}$-subobjects of the objects from $\mathcal{A}$. Dual notions: $\mathbf{Q}_{\mathcal{E}}(\mathcal{A})$, where $\mathcal{E} \subset \mathcal{E}pi$. Let $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and $l : \mathcal{C}_2\mathcal{V} \to \mathcal{L}$ be a coreflective functor and a reflective functor, respectively. We will denote:

$\mu\mathcal{K} = \{m \in \mathcal{M}ono |\ k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{L} = \{e \in \mathcal{E}pi |\ l(e) \in \mathcal{I}so\}$,

$\mathbb{K}$ - the class of nonzero coreflective subcategories,

$\mathbb{R}$ - the class of nonzero reflective subcategories,

$\mathbb{R}^s(\varepsilon\mathcal{L}) = \{\mathcal{R} \in \mathbb{R} | \mathcal{R} = \mathbf{S}_{\varepsilon\mathcal{L}}(\mathcal{R})\}$, $\mathbb{K}^s(\mu\mathcal{K}) = \{\mathcal{T} \in \mathbb{K} | \mathcal{T} = \mathbf{S}_{\mu\mathcal{K}}(\mathcal{T})\}$,

$\mathbb{R}_f(\varepsilon\mathcal{L}) = \{\mathcal{R} \in \mathbb{R} | \mathcal{R} = \mathbf{Q}_{\varepsilon\mathcal{L}}(\mathcal{R})\}$, $\mathbb{K}_f(\mu\mathcal{K}) = \{\mathcal{T} \in \mathbb{K} | \mathcal{T} = \mathbf{Q}_{\mu\mathcal{K}}(\mathcal{T})\}$,

$\mathbb{R}^s_f(\varepsilon\mathcal{L}) = \mathbb{R}^s(\varepsilon\mathcal{L}) \cap \mathbb{R}_f(\varepsilon\mathcal{L})$, $\mathbb{K}^s_f(\mu\mathcal{K}) = \mathbb{K}^s(\mu\mathcal{K}) \cap \mathbb{K}_f(\mu\mathcal{K})$.

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For $\mathcal{T} \in \mathbb{K}$ and $\mathcal{H} \in \mathbb{R}$ let also denote: $\mathbb{F}_g(\mathcal{T}) = \{\mathcal{K} \in \mathbb{K}^s(\mu\mathcal{T}) | \mathcal{T} \subset \mathcal{K}\}$, $\mathbb{F}_d(\mathcal{H}) = \{\mathcal{R} \in \mathbb{R}_f(\varepsilon\mathcal{H}) | \mathcal{H} \subset \mathcal{R}\}$.

Examples. 1. The reflector functor $id : \mathcal{C}_2\mathcal{V} \to \mathcal{C}_2\mathcal{V}$ belongs to the class $\mathbb{F}_g(\mathcal{H})$. (1^*. The reflector functor $h : \mathcal{C}_2\mathcal{V} \to \mathcal{H}$ belongs to the class $\mathbb{F}_d(\mathcal{H})$.)

2. $\mathbb{F}_g(\mathcal{T})$ is a closed class with respect to the intersection. (2^*. $\mathbb{F}_d(\mathcal{T})$ is a closed class with respect to the intersection.)

Theorem. *For any $\mathcal{H} \in \mathbb{R}$, the class $\mathbb{F}_d(\mathcal{H})$ is a commutative monoid (associative and with the unit).*

1^*. *For any $\mathcal{T} \in \mathbb{R}$ the class $\mathbb{F}_g(\mathcal{T})$ is a commutative monoid.*

2. *Let $\mathcal{H}$, $\mathcal{H}_1 \in \mathbb{R}$, $\mathcal{H} \subset \mathcal{H}_1$, $\mathbb{U}_d(\mathcal{H}, \mathcal{H}_1) = \{\mathcal{R} \in \mathbb{R}_f(\varepsilon\mathcal{H}) | \mathcal{H}_1 \subset \mathcal{R}\}$, and $\mathbb{F}_d(\mathcal{H}, \mathcal{H}_1)$ be the class of reflective functors corresponding to the elements of $\mathbb{U}_d(\mathcal{H}, \mathcal{H}_1)$. Then $\mathbb{F}_d(\mathcal{H}, \mathcal{H}_1)$ is a commutative submonoid of monoids $\mathbb{F}_d(\mathcal{H})$ and $\mathbb{F}_d(\mathcal{H}_1)$.*

2^*. *Let $\mathcal{T}$, $\mathcal{T}_1 \in \mathbb{K}$, $\mathcal{T} \subset \mathcal{T}_1$, $\mathbb{T}_g(\mathcal{T}, \mathcal{T}_1) = \{\mathcal{K} \in \mathbb{K}^s(\mu\mathcal{T}) | \mathcal{K} \subset \mathcal{T}_1$, and $\mathbb{F}_d(\mathcal{T}, \mathcal{T}_1)$ be the class of reflective functors corresponding to the elements of $\mathbb{T}_g(\mathcal{T}, \mathcal{T}_1)$. Then $\mathbb{F}_g(\mathcal{T}, \mathcal{T}_1)$ is a commutative submonoid of the monoids $\mathbb{F}_g(\mathcal{T})$ and $\mathbb{F}_g(\mathcal{T}_1)$.*

3. *Let $\mathbb{F}_{dv}(\mathcal{H})$ be the class of the reflector functors $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$, where $\mathcal{R}$ is an $\mathcal{E}$pi-variety and $\mathcal{H} \subset \mathcal{R}$. Then $\mathbb{F}_{dv}(\mathcal{H})$ is a commutative submonoid of the monoid $\mathbb{F}_d(\mathcal{H})$.*

3^*. *Let $\mathbb{F}_{gv}(\mathcal{T})$ be the class of the coreflector functors $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$, where $\mathcal{K}$ is a $\mathcal{M}$ono-covariety (the coreflective subcategory closed with respect to $\mathcal{M}$ono-subobjects, for example Σ, $\mathcal{C}_2\mathcal{V}$) and $\mathcal{T} \subset \mathcal{K}$. Then $\mathbb{F}_{gv}(\mathcal{T})$ is a commutative submonoid of the monoid $\mathbb{F}_g(\mathcal{T})$.*

4. *Let $\mathbb{F}_{dc}(\mathcal{H})$ be the class of the reflector functors $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$, where $\mathcal{R} \in \mathbb{R}_c$ and $\mathcal{H} \subset \mathcal{R}$. Then $\mathbb{F}_{dc}(\mathcal{H})$ is a commutative submonoid of the monoid $\mathbb{F}_d(\mathcal{H})$ (see the example 1.2).*

4^*. *Let $\mathbb{F}_{gc}(\mathcal{T})$ be the class of the coreflector functors $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$, where $\mathcal{K} \in \mathbb{K}_c$ and $\mathcal{T} \subset \mathcal{K}$. Then $\mathbb{F}_{gc}(\mathcal{T})$ is a commutative submonoid of the monoid $\mathbb{F}_g(\mathcal{T})$.*

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DEFORMATION QUANTIZATION OF THE ALGEBRA OF COLORED JACOBI DIAGRAMS

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Extending work of Habiro and Massuyeau (2020), an extended Kontsevich-LMO-type invariant of links in thickened surfaces $\Sigma_{g,n}$ is constructed, and its universality among finite-type invariants of such links is established. Combining these with results of Andersen, Mattes, and Reshetikhin (1998), a canonical deformation quantization of the Poisson algebra $\mathbf{A}$ of colored Jacobi diagrams (or equivalently, of chord diagrams on $\Sigma_{g,n}$) is obtained. Moreover, a combinatorial Moyal-Weyl-type formula is established. Since several skein algebras in knot theory are obtained as quotients of $\mathbf{A}$, this allows to study deformation quantization of some skein algebras. Specifying a quadratic Lie algebra $\mathfrak{g}$, $\mathbf{A}$ describes the Poisson algebra of algebraic functions on the character variety $Hom(\pi_1(\Sigma_{g,n}), G)/G$, where $\mathfrak{g} = Lie\, G$, and therefore we infer the existence of a canonical Poisson deformation for the latter, which we plan to investigate elsewhere.

GROUPOIDS OF ORDER THREE UP TO ISOMORPHISMS

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The list of all classical Bol-Moufang identities is given in [1]. We continue count the number of non-isomorphic groupoids of order three with some Bol-Moufang identities [2, 3, 4, 5].

There exist 45 non-isomorphic groupoids of order 3 with the identity F_{21}, $yx \cdot zx = (yx \cdot z)x$ from possible 314 groupoids.

There exist 53 non-isomorphic groupoids of order 3 with the identity F_{22}, $x(y \cdot zx) = x(y \cdot zx)$ from possible 271 groupoids.

Identity F_{23} $yx \cdot zx = y(xz \cdot x)$: up to isomorphism there exist 45 groupoids from possible 223 groupoids.

Identity F_{24} $yx \cdot zx = y(xz \cdot x)$: up to isomorphism there exist 46 groupoids from possible 260 groupoids.

Identity F_{25} $(yx \cdot z)x = (y \cdot xz)x$: up to isomorphism there exist 110 groupoids from possible 601 groupoids.

Identity F_{26} $(yx \cdot z)x = y(xz \cdot x)$: up to isomorphism there exist 44 groupoids from possible 215 groupoids.

Identity F_{27} $(yx \cdot z)x = y(x \cdot zx)$: up to isomorphism there exist 34 groupoids from possible 164 groupoids.

Identity F_{28} $(y \cdot xz)x = y(xz \cdot x)$: up to isomorphism there exist 63 groupoids from possible 344 groupoids.

Identity F_{29} $(y \cdot xz)x = y(x \cdot zx)$: up to isomorphism there exist 76 groupoids from possible 404 groupoids.

Identity F_{30} $y(xz \cdot x) = y(x \cdot zx)$: up to isomorphism there exist 252 groupoids from possible 1456 groupoids.

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ON A SPECIAL METHOD OF CONSTRUCTING TOPOLOGICAL QUASIGROUPS

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A non-empty set G is said to be a *groupoid* with respect to a binary operation denoted by $\cdot$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$ [1]. If the groupoid G is a topological space and the

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multiplication operation $(a, b) \to a \cdot b$ is continuous, then G is called a *topological groupoid.*

A groupoid $(G, \cdot)$ is called *paramedial* if it satisfies the identity $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$. A groupoid $(G, \cdot)$ is called *medial* if it satisfies the law $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$.

A groupoid $(G, \cdot)$ is called a *quasigroup* if for every $a, b \in G$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.

If a guasigroup G contains an element e such that $e \cdot x = x$ $(x \cdot e = x)$ for all x in G, then e is called a *left (right) identity* element of G and G is called a *left (right) loop*, ([1, p.9]).

We give a new method of constructing non-associative medial and paramedial topological quasigroups.

The results established here are related to the work in ([2,3]).

Theorem 1. *Let $(G, +, \tau)$ be a commutative topological group where G is not a singleton. For (x_1, y_1) and (x_2, y_2) in $G \times G$ define*

$$(x_1, y_1) \circ (x_2, y_2) = (x_1 + y_1 + x_2 - y_2, -y_1 - y_2).$$

Then $(G \times G, \circ, \tau_G)$, relative to the product topology τ_G, is a paramedial, non-medial and non-associative topological quasigroup. Moreover, if (G, τ) is $T_i -$ space, then $(G \times G, \tau_G)$ is $T_i -$ space too, where $i = 1, 2, 3, 3.5$.

Theorem 2. *Let $(G, +, \tau)$ be a commutative topological group where G is not a singleton. For (x_1, y_1) and (x_2, y_2) in $G \times G$ define*

$$(x_1, y_1) \circ (x_2, y_2) = (x_1 - 2y_2 + x_2, y_1 + y_2).$$

Then $(G \times G, \circ, \tau_G)$, relative to the product topology τ_G, is a medial, non-paramedial, non-associative topological quasigroup with right identity. Moreover, if (G, τ) is $T_i -$ space, then $(G \times G, \tau_G)$ is $T_i -$ space too, where $i = 1, 2, 3, 3.5$.

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ON CERTAIN GENERALIZED ORE TYPE MONOID RINGS

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Given a non-commutative associative unital ring A, a multiplicative monoid G, and a family σ of mappings $A \to A$ satisfying some suitable properties (axioms), a monoid algebra $A < G >$ was introduced in [1] (and also [2]). The construction made generalizes, in a natural way, the classical Ore extensions [3], differential (but also difference) polynomial rings [4], [5], related results of Smits [6], as well as others. In certain special situations, but important in applications, such as, in particular, the case of families σ of nilpotent mappings, it was possible to describe completely the structure of the monoid algebras in force. The obtained results can be actually applied in the study of Weyl algebras [2], [7], in theory of so-called skew-polynomial rings [2], [7], and others related domains.

Recently, Nystedt, Oinert and Richter [8] have proposed a slightly more general construction of monoid rings by considering the case of a non-commutative ring A. It is noteworthy that in [8], for the case of commutative monoids, necessary and sufficient conditions are given for the corresponding monoid rings to be simple.

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ON RECURSIVE DIFFERENTIABILITY OF SOME QUASIGROUPS PROLONGATIONS

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By a prolongation of a finite quasigroup Q, we meen a method of construction that extends Q to a new finite quasigroup. R. Bruck proposed a construction of prolongations for idempotent quasigroups and V. Belousov considered an analogues method using an arbitrary transversal [1,2]. Other constructions of prolongations, or new ideas for their generalization, have been proposed by G. Belyavskaya, I. Deriyenko and W. Dudek, V. Shcherbacov.

The recursive 1-differentiability of prolongations is studied in the present work. A binary quasigroup is called recursively 1-differentiable if the groupoid $(Q, \circ)$, where $x \circ y = y \cdot (x \cdot y)$, $\forall x, y \in Q$, is a quasigroup. At present it is known that there exist recursively 1-differentiable binary quasigroups of order q, for every q, excepting $1, 2, 6$, and possibly $14, 18, 26$ [3]. Also it is an open problem to find the maximum r such that a finite binary (or n-ary) quasigroup is recursively r-differentiable [4]. We give necessary and sufficient conditions when the prolongations, obtained using Bruck and, respectively, Belousov constructions, are recursively 1-differentiable quasigroups.

Theorem 1. *Let $(Q, \cdot)$ be a finite quasigroup, such that $\theta : Q \mapsto Q, \theta(x) = x \cdot x$, is a bijection and $x \neq \theta(x)$, $\forall x \in Q$. Then the prolongation $(Q', \circ)$, obtained by Bruck's method, where $Q' = Q \cup \{\xi\}$, is recursively 1-differentiable if and only if the following conditions hold:*

1. *$\{f_y | y \in Q\} = Q$, where f_y is the left local unit of y in $(Q, \cdot)$;*
2. *$y \to y \cdot \theta(y)$ is a bijection in $(Q, \cdot)$;*
3. *$\{\theta(x), \theta^2(x), y \cdot xy \mid y \neq x, f_y \neq x, y \in Q\} = Q$, $\forall x \in Q$.*

Theorem 2. *Let $(Q, \cdot)$ be a finite quasigroup, with a complete substitution θ and let $\theta'(x) = x \cdot \theta(x), \forall x \in Q$. Then the prolongation $(Q', \circ)$, obtained by Belousov's method, for $k = 1$, is recursively 1-differentiable if and only if the following conditions hold:*

1. *$\theta^{-1}y \neq \theta'(\theta^{-1}(y)), \forall y \in Q$, and $\{\theta^{-1}(y)/y \mid y \in Q\} = Q$;*
2. *$y \to y \cdot \theta'(\theta^{-1}(y))$ is a bijection on Q;*

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3. $\{\theta'(\theta(x)), y\cdot xy,\ \theta'(\theta^{-1}(\theta'(x)))|y \neq \theta(x\cdot y), y \neq \theta(x), y \in Q\} = Q, \forall x \in Q.$

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HYPERBOLIC MANIFOLDS BUILT ON THE GEOMETRIES OF THEIR CUSPS OR SUBMANIFOLDS

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We describe geometric methods that allow to build and investigate hyperbolic manifolds of dimensions 3, 4 and 5, with certain predefined properties, such as cusps geometry, the geometry of a totally geodesic submanifold, etc. Some result of these constructions look like as a generalization of 2-dimensional pants. Also we use these examples and methods of metric reconstruction to obtain non-face-to-face incidence schemes for fundamental polyhedra. As a result, new manifolds and some exotic tilings on universal coverage are obtained. The talk will be focused on the transfer of methods of discrete geometry to topology and vice versa.

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ON TWO COUNTABLE SERIES OF "CLOSE" HYPERBOLIC MANIFOLDS WITH FINITE VOLUME

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The communication is devoted to the construction of two countable series of hyperbolic manifolds of finite volume. For every k where $k = 2, 3, ...$, we construct a compact and a noncompact three-dimensional hyperbolic manifolds such that their fundamental groups differ only in one generator whereas the other generators of these fundamental groups metrically coincide.

First give the construction of the corresponding polytopes in the hyperbolic space H^3.

In a hyperbolic plane ω we consider a regular $4k$-gon with all right angles where $k = 2, 3, ...$ It is easy to prove its existense. Through sides of this polygon we draw planes $\alpha_1, \alpha_2, ..., \alpha_{4k}$ ortogonal to the plane ω. Let $a_i = \alpha_i \cap \alpha_{i+1}, i = 1, 2, ..., 4k-1$, and $a_{4k} = \alpha_{4k} \cap \alpha_1$ be the straight lines of the intersection of these planes, the lines are orthogonal to the plane ω. On the straight lines a_i we intersept segments of length h on both sides of the plane ω and through their ends draw planes, β_i above the plane ω and γ_i below the plane ω, orthogonal to the straight lines a_i. It is not difficult to show that we can choose the length so that any plane β_i intersects the neighboring planes by the angle of $\pi/2$ and the same holds for planes γ_i. Let b_i be the straight lines of the intersection of the planes β_i and c_i be the intersections of the planes γ_i. Then the straight lines b_i form a hyperbolic bandle of straight lines as well as the lines c_i form a hyperbolic bandle. Let δ_1 be the plane orthogonal to the straight lines c_i and δ_2 be the plane orthogonal to all the lines b_i where $i = 1, 2, ..., 4k$. Then the polytope R_{4k} formed by the planes α_i, β_i, γ_i, δ_1, δ_2 is bounded. If we take the planes t_1 and t_2 protective to the bundles of straight lines c_i and b_i respectively, then we obtain a polytope T_{4k}, which is unbounded but its volume is finite.

For polytopes R_{4k} and T_{4k} we show motions that identify their faces, these motions are generators of discrete groups Γ_{4k} and P_{4k}, and we prove that these groups are torsion-free. All the generators of these groups metrically coincide except for motions that identify the faces δ_1 and δ_2 in the group Γ_{4k} and the faces t_1 and t_2 in the group P_{4k}.

SOME PROPERTIES OF IG-QUASIGROUPS

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We investigate quasigroups isotopic to groups ($IG-$quasigroups). A groupoid $(Q, \circ)$ is called isotopic to the groupoid $(Q, \cdot)$, if there exist three permutations $\alpha, \beta, \gamma \in S(Q)$ such that for any $x, y \in Q$ it is satisfied $x \circ y = \gamma^{-1}(\alpha x \cdot \beta y)$. If $(Q, \cdot)$ is a quasigroup then $(Q, \circ)$ is also a quasigroup ([1], p. 14). If $\gamma = \varepsilon$ then $(Q, \circ)$ is calld a principal isotop of $(Q, \cdot)$. It is known that any isotope is isomorphic to some principal isotope. We follows research method from [2], when studying quasigroups in general isotopic to groups. In the paper [3] it was proved:

Lemma [3]. *Let $(Q, +, 0)$ be a group with zero 0, $\alpha, \beta \in S(Q)$, and $(Q, \circ)$ an isotope of the form: $x \circ y = \alpha x + \beta y$. Then there exists $a \in Q$, $\alpha_1, \beta_1 \in S_0(Q)$ such that $\alpha_1 0 = 0$, $\beta_1 0 = 0$, $a = 0 \circ 0$ and $x \circ y = \alpha_1 x + a + \beta_1 y$ for any $x, y \in Q$.*

Definition. *Let $(Q, +, 0)$ be a group with zero 0.*
The quadruple $((Q, +, 0), \alpha_1, \beta_1, a)$, $\alpha_1, \beta_1 \in S_0(Q)$ is called the $IG-$form of the principal isotope $(Q, \circ), x \circ y = \alpha x + \beta y, \alpha, \beta \in S(Q), x, y \in Q$, if $x \circ y = \alpha_1 x + a + \beta_1 y$, $\alpha_1 0 = 0 = \beta_1 0$, $a = 0 \circ 0$.

Theorem. *Let $(Q, +, 0)$ be a group with zero 0, ε is identical permutation on Q and $((Q, +, 0), \alpha_1, \beta_1, a)$, $x \circ y = \alpha_1 x + a + \beta_1 y$, $\alpha_1, \beta_1 \in S_0(Q)$, $x, y \in Q$ is its $IG-$form. Than:*

a) $(Q, \circ)$ is commutative, iff $(Q, +, 0)$ is commutative and $\alpha_1 = \beta_1$,
b) $(Q, \circ)$ is medial, iff $\alpha_1, \beta_1 \in Aut(Q, +)$ and $\alpha_1\beta_1 = \beta_1\alpha_1$,
c) $(Q, \circ)$ is a group, iff $\alpha_1 = \beta_1 = \varepsilon$,
d) $(Q, \circ)$ is idempotent quasigroup, iff $a = 0$ and $\alpha_1 + \beta_1 = \varepsilon$,
e) $(Q, \circ)$ is left (right) symmetric quasigroup, iff $(Q, +, 0)$ is abelian, and $\beta_1 = \varepsilon$, $(\alpha_1 = \varepsilon)$,
f) $(Q, \circ)$ is a symmetric quasigroup, iff $(Q, +, 0)$ is abelian, and $\alpha_1 = \beta_1 = \varepsilon$,
g) $(Q, \circ)$ is a left distributive quasigroup, iff $a = 0$, $\beta_1 \in Aut(Q, +)$, $\alpha_1\beta_1 = \beta_1\alpha_1$,$\alpha_1 + \beta_1 = \varepsilon$,
h) $(Q, \circ)$ is a right distributive quasigroup, iff $a = 0$, $\alpha_1 \in Aut(Q, +)$, $\alpha_1\beta_1 = \beta_1\alpha_1$,$\alpha_1 + \beta_1 = \varepsilon$,
i) $(Q, \circ)$ is a distributive quasigroup, iff $a = 0$, $\alpha_1, \beta_1 \in Aut(Q, +)$, $\alpha_1\beta_1 = \beta_1\alpha_1$,$\alpha_1 + \beta_1 = \varepsilon$, j)

[7] *Speaking author*: V. Izbash

k) $(Q, \circ)$ *is a Steiner quasigroup, iff* $(Q, +, 0)$ *is abelian,* $\alpha_1 = \beta_1 = \varepsilon$*, and* $3x = 0$ *for all* $x \in Q$*.*

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FROBENIUS GROUPS AND ONE-SIDED S-SYSTEMS

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Definition. *The transitive irregular permutation group* G *acting on a set* E *is called a* ***Frobenius group****, if* $St_{ab}(G) =< \mathbf{id} >$ *for any* $a, b \in E,\ a \neq b$*.*

Let G be an arbitrary Frobenius group of permutations on some set E and $0,\ 1$ be two distinct distinguished elements from E. As the group G is transitive on the set E, then there exists a set of n permutations $P = \{\sigma_x\}_{x \in E}$ such that $\sigma_x(0) = x \quad \forall x \in E$, and $\sigma_0 = \mathbf{id}$. Let's define the operation $(x,\ a,\ y)$ as follows:

$$(x, 0, y) \stackrel{def}{=} x, \quad (x, 1, y) \stackrel{def}{=} y,$$

$$\begin{array}{ll} \forall a \in E^*,\ a \neq 0,\ 1: & (x, a, y) = z \quad \stackrel{def}{\Leftrightarrow} \quad z = \alpha(y), \\ \text{where}\ \ \alpha \in G, & \alpha(x) = x, \quad \beta(1) = (\sigma_x^{-1} \alpha \sigma_x)(1) = a. \end{array} \tag{1}$$

Lemma 1. *The operation* $(x,\ a,\ y)$ *satisfies the following properties:*

1. $(0, a, 1) = a, \quad (x, a, x) = x;$
2. $\forall a, b \in E^*: \qquad (x, a, (x, b, y)) = (x, c, y)$ *for some* $c = c(a, b) \in E^*$*;*
3. *if* n *is finite, then any binary operation* $A_a = (x, a, y)$ *is a quasigroup* $(a \in E^*)$*.*

Lemma 2. *The mappings*

$$\varphi_{b,a}\,(x) = (b,\, a,\, x)\,, \quad b \in E, \quad a \in E^* - \{0\};$$

$$\psi_{b,\,a,\,d}\,(x) = \Big(b,\, a,\, \Big(d,\, a^{(\,-1)},\, x\Big)\Big)\,, \quad b,\, d \in E, \quad a \in E^* - \{0\};$$

are permutations on the set E*.*

Lemma 3. *The following statements are true:*

1. *The permutation* $\varphi_{b,\,a} \quad (b \in E, \quad a \in E^* - \{0,\,1\})$ *has one and only one fixed element* b;

2. *The permutation* $\psi_{b,\,a,\,d} \quad (b,\,d \in E, \quad a \in E^* - \{0,\,1\})$ *is a fixed-point-free permutation, if* $b \neq d$;

3. *The set of permutations*

$$T = \{\psi_{b,\,a_0,\,0} | \, b \in E, \quad a_0 \ \ is\ a\ fixed\ element\ from \ \ E^* - \{0,\,1\}\},$$

is transitive on the set E.

Lemma 4. *Let permutations* $\varphi_{b,a}$ *and* $\psi_{b,a,d}(b, d \in E, \quad a \in E^* - \{0\})$ *form a group* G *under the natural product of permutations. Then* G *is a Frobenius group.*

By means of last Lemmas we obtain that there exist normal subgroups, consisting of fixed-point-free permutations and the identity permutation, in the finite Frobenius groups, which are groups of cell permutations of the right S-system.

Theorem. *Let the set* E *be a finite one. If the conditions of last Lemma hold, then the set of fixed-point-free permutations*

$$T = \{\psi_{b,\,a_0,\,0} | \, b \in E, \quad a_0 \ \ is\ any\ fixed\ element\ from \ \ E^* - \{0,\,1\}\}$$

with the identity permutation $\mathbf{id} = \psi_{0,a_0,0}$ *is a normal subgroup in the Frobenius group* G.

ON SOME PARTICULARITIES OF THE GENERAL THEORY OF DISCRETE W_p-SYMMETRY GROUPS

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The description of the symmetry of some real crystal structures led to the W-symmetry idea of Koptsik and Kotsev [1]. The general theory of W-symmetry groups was elaborated in the nineties of the XX century [2-8]. Now we will analyze only some specific particularities of discrete W_p-symmetry groups.

We will highlight only the following specific aspects:

1. W is the Cartesian product of isomorphic copies of the initial transitive group of substitutions P, which are indexed with elements g of G, i.e. $W = \prod_{g_i \in G} P^{g_i}$, where $P^{g_i} \cong P$.

2. The set of transformations of W_p-symmetry of the "indexed" geometrical figure $F^{(N)}$ forms a group $G^{(W_p)}$ with the operation $g_i^{(w_i)} \circ g_j^{(w_j)} = g_k^{(w_k)}$, where $g_k = g_i g_j$, $w_k = w_i^{g_j} w_j$ and $w_i^{g_j}(g_s) = w_i(g_j g_s)$.

3. The W_p-symmetry groups with the initial substitution group P and the classical symmetry generating group G are subgroups, which verify certain specific conditions, of the left simi-direct products of group W with the group $\underset{\leftarrow}{G}$, accompanied with a fixed isomorphism $\varphi : G \to AutW$ by the rule $\varphi(g) = \overleftarrow{g}$, where $\overleftarrow{g} : w \mapsto w^g$.

4. If the group W is finite, then: a) $V^g = wVw^{-1}$ for all components g and w of transformations $g^{(w)}$ of group $G^{(W_p)}$, where V is the subgroup of W-identical transformations in $G^{(W_p)}$; b) any element of the class of residues Hg is combined in pairs with each element of residues class wV, the elements of different classes Hg_i and Hg_j are combined with the elements of different classes w_iV and w_jV, respectively, where H is the subgroup of classical symmetry in $G^{(W_p)}$.

5. The equivalent groups of W_p-symmetry have: a) the same generating group, b) the same initial group P of substitutions, c) the conjugated symmetry subgroups, d) the conjugated subgroups of W-identical transformations, e) the equivalent subgroups of P-symmetry.

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ON THE RING OF CONTINUOUS ENDOMORPHISMS OF AN LCA GROUP

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Given a locally compact abelian group X, let $E(X)$ denote the set of all continuous endomorpgisms of X. Then $E(X)$ can be turned into a complete Hausdorff topological ring by endowing it with the operetions of pointwise addition and composition of maps, and with the compact-open topology.

In our communication, we will present some results concerning the interplay between the properties of X and those of $E(X)$.

GENERALIZED ASSOCIATIVITY AND TARSKI QUASIGROUPS

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Necessary definitions can be found in [1, 4].

Definition. *A quasigroup $(Q, \cdot)$ is a T-quasigroup if and only if there exists an abelian group $(Q, +)$, its automorphisms φ and ψ and a fixed element $a \in Q$ such that $x \cdot y = \varphi x + \psi y + a$ for all $x, y \in Q$ [2]. If in a T-quasigroup $\varphi\psi = \psi\varphi$, then it is called a medial quasigroup.*

In the quasigroup case the Schröder identity $(y \cdot z)\backslash x = z(x \cdot y)$ is equivalent to the following identity $(y \cdot z) \cdot (z \cdot (x \cdot y)) = x$ [3, 4].

Theorem 1. *In a T-quasigroup $(Q, \cdot)$ of the form $x \cdot y = \varphi x + \psi y$ the Schröder identity of generalized associativity is true if and only if $\varphi = \psi^{-2}$, $\varepsilon = \varphi^7$, $\varepsilon = \psi^{14}$, $\varphi\psi z + \psi\varphi z = 0$.*

We give the following example of a medial quasigroup with the identity of generalized associativity:

[8] *Speaking author*: P.A. Radilov

$\circ$	1	2	3	4	5	6	7	8
1	1	4	8	5	3	2	6	7
2	3	2	6	7	1	4	8	5
3	6	7	3	2	8	5	1	4
4	8	5	1	4	6	7	3	2
5	7	6	2	3	5	8	4	1
6	5	8	4	1	7	6	2	3
7	4	1	5	8	2	3	7	6
8	2	3	7	6	4	1	5	8

Here we concentrate over medial quasigroups with the following Tarski identity: $x(y(zx)) = zy$ [5].

Theorem 2. *In a medial quasigroup $(Q, \cdot)$ of the form $x \cdot y = \varphi x + \psi y$ the Tarski identity is true if and only if $\varphi = \varepsilon$, $\psi = I$.*

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MEDIAL i-QUASIGROUPS

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i-Quasigroups have been studied by I. A. Florja and N. N. Didurik [1].

Definition. A quasigroup with the i-identity

$$x(xy \cdot z) = y(zx \cdot x) \tag{1}$$

[9] *Speaking author*: I.N. Radilova

is called an i-quasigroup.

This identity is similar to the Moufang identity [2, 3, 4] but it is not the same identity. Quasigroups with this identity form a new class of quasigroups [1]. Here we study in main medial i-quasigroups.

Necessary definitions can be found in [2, 3, 4].

Examples of i-quasigroups (including i-quasigroups of order 5) are given in [1].

Theorem 1. *In a medial quasigroup* $(Q, \cdot)$ *of the form* $x \cdot y = \varphi x + \psi y$ *the* i*-identity is true if and only if* $\psi = \varphi^2$, $\varphi^4 = \varepsilon$.

Example. The quasigroup $(Z_{15}, \circ)$, where $x \circ y = 2 \cdot x + 4 \cdot y \pmod{15}$, is a medial i-quasigroup.

Theorem 2. *Suppose that we have a commutative (abelian) group* C*, such that the group* $Aut(C)$ *contains an element* φ *of order* 4*. Then medial quasigroup of the form* $x \circ y = \varphi x + \varphi^2 y$ *is a medial* i*-quasigroup.*

Notice, there exist non-medial i-quasigroups.

$*$	0	1	2	3	4	5	6	7
0	2	3	0	1	5	4	7	6
1	0	5	6	3	4	1	2	7
2	3	2	1	0	7	6	5	4
3	1	7	4	2	6	0	3	5
4	5	0	3	6	2	7	4	1
5	7	1	2	4	3	5	6	0
6	4	6	5	7	0	2	1	3
7	6	4	7	5	1	3	0	2

Indeed, $(5 * 2) * (4 * 7) = 2 * 1 = 2$; $(5 * 4) * (2 * 7) = 3 * 4 = 6$.

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PARASTROPHES OF TERNARY QUASIGROUPS AND THEIR ORTHOGONALITY

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An n-ary groupoid (Q, A) is called an n-ary quasigroup if each of the elements $x_1, x_2, ..., x_{n+1}$ in the equality $A(x_1, x_2, ..., x_n) = x_{n+1}$ is uniquely determined by the remaining n. The operation ${}^{\sigma}A$, defined by the equivalence

$$A(x_1, x_2, ..., x_n) = x_{n+1} \Leftrightarrow^{\sigma} A = (x_{\sigma 1}, x_{\sigma 2}, ..., x_{\sigma n}) = x_{\sigma(n+1)},$$

where σ is a substitution of degree $n+1$, is called a parastrophe of (Q, A).

It is proved in [1] that the number of distinct parastrophes of a binary quasigroup is always a divisor of 6. According to [2], the number of distinct parastrophes of a ternary quasigroup is a divisor of 24. The classes of quasigroups with a fixed number of pairwise distinct parastrophes are considered by many authors (see [2-5]).

The classes of distinct parastrophes, orthogonality and parastrophic orthogonality of ternary quasigroups are considered in the present work. We found necessary and sufficient conditions such that:

(i) a linear ternary quasigroup has a given type of parastrophic orthogonality;

(ii) the set of all 6 main parastrophes of a linear ternary quasigroup is orthogonal.

Lemma. *The ternary quasigroups A, B, C, defined on a set Q, are orthogonal if and only if the operations $K_1(x, y, z) = B(x, y, {}^{(34)}A(x, y, z))$ and $D(x, y, z) = C(x, {}^{(24)}K_1(x, y, z), {}^{(34)}A(x, {}^{(24)}K_1(x, y, z), z))$ are, respectively, $2-$ and $1-$invertible.*

Theorem. *There exist self-orthogonal ternary finite quasigroups of order q, for every odd q, such that $(q, 3) = 1$.*

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ON MULTIPLICATION GROUPS OF BOL LOOPS

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The (total) multiplication group of a quasigroup Q is the group generated by all (left, right and middle) left and right translations of Q. Natural transformations of quasigroups (loops) are parastrophy (inversion), isotopy and isostrophy (the product of a parastrophy and an isotopy)(see [1-2]). It is known that multiplication (total multiplication) groups of isotopic (isostrophic) loops are isomorphic (see [3]). Invariant properties under the isotopy of loops are called universal properties. (Left, right) middle Bol loops are loops with universal (left inverse, right inverse) anti-automorphic inverse property. Moreover, middle Bol loops are isostrophic to the left and to the right Bol loops. Connections between multiplication and total multiplication groups of loops (in particular, of left, right and middle Bol loops) are considered in the present talk. A characterization of the commutative middle Bol loops is given.

This is a joint work with A. Drapal.

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ON DISCERNING DELONE CLASSES OF 3-ISOHEDRAL SPHERICAL TILINGS

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For any discrete isometry group of the sphere, the splitting procedure can be applied to fundamental 2-isohedral tilings of the sphere with disks [1]. As a result we obtain all the fundamental 3-isohedral tilings of the sphere with disks for this group. However some Delone classes may appear several times when doing such a procedure. So in order to get the full classification we have to do the second step that consists in discerning which tilings belong to the same Delone class. But there is no convenient method for doing that. To gain the goal we establish the ordering in the list of obtained tilings and then compare neighboring ones.

Each polygonal disk in the tiling has a type written as $n_1.n_2 \dots n_r$ where n_1, n_2, $\dots$,n_r refer to the valencies of the vertices in the tiling. Among different possibilities to write the type we choose the one with the smallest number $n_1 n_2 \dots n_r$. For 3 transitivity classes of polygonal disks in the tiling, we write together the 3 types, beginning with the smallest number of edges. Remark that such a type does not determine the whole tiling of the sphere.

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II. ANALYSIS, DIFFERENTIAL EQUATIONS AND DYNAMICAL SYSTEMS

AVERAGING OF MULTI-FREQUENCY FIRST APPROXIMATION SYSTEMS WITH DELAY AND THEIR APPLICATION

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A system of differential equations of the form

$$\begin{aligned} \frac{da}{d\tau} &= X_0(\tau, a_\Lambda) + \varepsilon X_1(\tau, a_\Lambda, \varphi_\Theta), \\ \frac{d\varphi}{d\tau} &= \frac{\omega(\tau, a)}{\varepsilon} + Y_0(\tau, a_\Lambda) + Y_1(\tau, a_\Lambda, \varphi_\Theta), \end{aligned} \tag{1}$$

where $\tau \in [0, L]$, $a \in D \subset R^n$, $\varphi \in T^m$ - torus in R^m, $0 < \varepsilon \leq \varepsilon_0 \ll 1$, $\Lambda = (\lambda_1, \ldots, \lambda_p)$, $\Theta = (\theta_1, \ldots, \theta_q)$, $\lambda_i, \theta_j \in (0, 1)$, $a_{\lambda_i}(\tau) = a(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$ is considered. Multipoint conditions are set for the system (1)

$$\sum_{\nu=1}^{r} \alpha_\nu a(\tau_\nu) = d_1, \quad \sum_{\nu=1}^{r} \beta_\nu \varphi(\tau_\nu) = d_2, \tag{2}$$

where $0 \leq \tau_1 < \tau_2 < \cdots < \tau_r \leq L$.

The system (1), when $X_0 = Y_0 = 0$ and $\omega = \omega(\tau)$ with integral and multipoint conditions was studied in [1,2].

Much simpler than the system (1) is the system of the form

$$\begin{aligned} \frac{d\overline{a}}{d\tau} &= X_0(\tau, \overline{a}_\Lambda) + \varepsilon X_1(\tau, \overline{a}_\Lambda), \\ \frac{d\overline{\varphi}}{d\tau} &= \frac{\omega(\tau, \overline{a})}{\varepsilon} + \varepsilon Y_0(\tau, \overline{a}) + \varepsilon Y_1(\tau, \overline{a}_\Lambda) \end{aligned} \tag{3}$$

with the conditions of the form (2), averaged over fast variables $\varphi_{\theta_1}, \ldots, \varphi_{\theta_q}$.

The component $\overline{a}(\tau; \overline{y}, \varepsilon)$, where $\overline{a}(0; \overline{y}, \varepsilon) = \overline{y}(\varepsilon)$, of the solution of the system (3) is already independent of $\overline{\varphi}(\tau, \varepsilon)$, which greatly simplifies the problem.

[10] *Speaking author*: Ya. Bihun

The existence of a solution to the problem (1), (2) is proved if there is a unique solution of the averaged problem. Also specified are the conditions under which

$$\|a(\tau; y, \psi, \varepsilon) - \overline{a}(\tau; \overline{y}, \varepsilon)\| + \varepsilon\|\varphi(\tau; y, \psi, \varepsilon) - \overline{\varphi}(\tau; \overline{y}, \overline{\psi}, \varepsilon)\| \leq c_1\varepsilon^{1+\alpha}, \quad (4)$$

where $0 < \alpha \leq (mq)^{-1}$, $a(0; y, \psi, \varepsilon) = y(\varepsilon)$, $\varphi(0; y, \psi, \varepsilon) = \psi(\varepsilon)$, $\overline{\varphi}(0; \overline{y}, \overline{\psi}, \varepsilon) = \overline{\psi}(\varepsilon)$, while $\|y - \overline{y}\| + \varepsilon\|\psi - \overline{\psi}\| \leq c_2\varepsilon^{1+\alpha}$, $c_1 > 0$, $c_2 > 0$ and do not depend on ε. The evaluation of (4) is performed for each $\varepsilon \in (0, \varepsilon^*], \varepsilon^* \leq \varepsilon_0$, and all $\tau \in [0, L]$.

The method given in the monograph [3] was used to prove the results.

The obtained result was applied to research the influence of multifrequency disturbances on the dynamics of predator-prey population interaction.

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POISSON STABLE MOTIONS AND GLOBAL ATTRACTORS OF SYMMETRIC MONOTONE NONAUTONOMOUS DYNAMICAL SYSTEMS

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Suppose that the reaction rates depend on time

$$S' = \Gamma f(t, S), \quad (1)$$

where $f(t, S)$ is almost periodic (respectively, quasi-periodic, Bohr almost periodic, automorphic, Birkhoff recurrent, Levitan almost periodic, Bebutov almost recurrent, Poisson stable) in t. Choosing $\mu \in \mathbb{R}^m_+$ and using the reaction coordinates $x : S = \mu + \Gamma x$, we transform (1) into a system in the reaction coordinates:

$$x' = F_\mu(t, x) := f(t, \mu + \Gamma x) \quad (2)$$

evolving on the state space X_μ.

Assume that the function $F_\mu \in C(\mathbb{R} \times X_\mu, \mathbb{R}^n)$ is regular. Let $\phi(t, v, G)$ denote the solution of

$$y' = G(t, y) \ (G \in H(F_\mu)) \tag{3}$$

passing through v at $t = 0$. Then it enjoys positive translation invariance:

$$\phi(t, \xi + \lambda v, G) = \varphi(t, v, G) + \lambda v, \ \forall \ \xi \in X_\mu, \ \lambda \in \mathbb{R} \text{ and } G \in H(F_\mu), \tag{4}$$

where $H(F_\mu)$ is the hull of F_μ, i.e., the cocycle φ admits a group of symmetry $\{T_\lambda | \ \lambda \in \mathbb{R}\}$ ($T_\lambda x := x + \lambda v$ for any $x \in X_\mu$ and $\lambda \in \mathbb{R}$).

A function $f \in C(\mathbb{R} \times W, \mathbb{R}^n)$ is said to be *strongly Poisson stable* if every function $g \in H(f)$ is Poisson stable and the set $H(f)$ is minimal.

Theorem. *Suppose that the following conditions hold:*

1. $\mu \in \mathbb{R}^m$ is such that the system (2) is strongly monotone;

2. the matrix Γ has rank exactly $n-1$ whose kernel is spanned by a strongly positive vector v;

3. for any $G \in H(F_\mu)$ all forward solutions of equation (3) are bounded;

4. the function $f \in C(\mathbb{R} \times X_{(\mu,\Gamma)}, \mathbb{R}^n)$ is stationary (respectively, τ-periodic, quasi-periodic, Bohr almost periodic, almost automorphic, recurrent in the sense of Birkhoff, strongly Poisson stable) in time $t \in \mathbb{R}$.

Then for any $U_0 \in X_{(\mu,\Gamma)}$ the following statements hold:

1. the set $\omega_{(U_0,f)} \bigcap X_f$ consists of a single point $p_0 = (V_0, f)$;

2. $\varphi(t, V_0, f)$ is a stationary (respectively, τ-periodic, quasi-periodic, Bohr almost periodic, almost automorphic, recurrent in the sense of Birkhoff, strongly Poisson stable) solution of equation (2);

3. $\lim\limits_{t \to +\infty} |\varphi(t, U_0, f) - \varphi(t, V_0, f)| = 0$, i.e., $\varphi(t, u_0, f)$ is asymptotically stationary (respectively, asymptotically τ-periodic, asymptotically quasi-periodic, asymptotically Bohr almost periodic, asymptotically almost automorphic, asymptotically recurrent in the sense of Birkhoff, asymptotically strongly Poisson stable).

CENTER CONDITIONS FOR A CUBIC SYSTEM WITH TWO HOMOGENEOUS INVARIANT STRAIGHT LINES AND EXPONENTIAL FACTORS

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We consider the cubic system of differential equations

$$\begin{aligned} \dot{x} &= y + ax^2 + (g-b)xy + fy^2 + (u+l)x^3 + mx^2y + \\ &\quad + (u+q)xy^2 + ry^3, \\ \dot{y} &= -(x + gx^2 + (f-a)xy + by^2 + (v-r)x^3 + qx^2y + \\ &\quad + (v-m)xy^2 + ly^3), \end{aligned} \tag{1}$$

where the coefficients and the variables in (1) are assumed to be real. The origin $O(0,0)$ is a singular point which is a center or a focus (fine focus) for (1). We determine the center conditions for cubic differential system (1) assuming that the system has invariant straight lines and exponential factors. It is easy to verify that the system (1) has the invariant straight lines $x \pm iy = 0$, $i^2 = -1$.

The problem of the center was solved for system (1) with: one invariant straight line $1 + Ax + By = 0$ in [1], two invariant straight lines of the form $C + Ax + By = 0$ in [2], one invariant conic $a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{10}x + a_{01}y + 1 = 0$ in [3]. By using the method of Darboux integrability and rational reversibility, the center conditions were found for system (1) in [4].

In this talk we consider the following problems:

(i) determine the cubic systems (1) which have exponential factors of the form $F = \exp\left(g(x,y)\right)$, where $g(x,y)$ is a real polynomial with degree$(g) \leq 2$.

(ii) determine the cubic systems (1) which have exponential factors of the form $\Phi = \exp\left(\frac{g(x,y)}{x^2+y^2}\right)$, where $g(x,y)$ is a real polynomial with deg $g \leq 2$.

(iii) for these classes of cubic systems find the center conditions.

It is proved [5] that the cubic system (1) with an exponential factor F has a center at the origin $O(0,0)$ if and only if the first five Lyapunov quantities vanish and the cubic system (1) with an exponential factor Φ has a center at the origin $O(0,0)$ if and only if the first two Lyapunov quantities vanish.

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RIEMANN AND PROJECTIVE SPACES IN THEORY OF THE FIRST AND SECOND ORDER ODE's

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The second order ODE's of the form

$$y_{xx} + a_1(x,y)y_x^3 + 3\,a_2(x,y)y_x^2 + 3\,a_3(x,y)y_x + a_4(x,y) = 0, \qquad (1)$$

where the coefficients $a_i(x,y)$ are arbitrary smooth functions, conserve their form under non degenerated changes of variables $x = x(u,v)$, $y = y(u,v)$. We shall use this type of equations to study the properties of solutions of polynomial systems $dy_s = Q_n(x(s), y(s), b_i), dx_s = P_n(x(s), y(s), b_i)$ with parameters b_i.

Theorem 1. [1] *The equations of geodesical lines of four-dimensional Riemann metric with coordinates* $[x, y, z, t]$

$$ds^2 = (za_3(x,y) - ta_4(x,y))\,dx^2 + dxdz + dydt +$$

$$(za_2(x,y) - ta_3(x,y))\,dxdy + (za_1(x,y) - ta_3(x,y))\,dy^2$$

for coordinates $x(s), y(s)$ *have the form*

$$y_{ss} + a_4x_s^2 + 2\,a_3x_sy_s + a_2y_s^2 = 0,\ \ x_{ss} - a_3x_s^2 - 2\,a_2x_sy_s - a_1y_s^2 = 0,$$

which are equivalent to the equation (1) and they are applied then to construct its solutions [1].

To study the projective properties of the equation (1) the following results are applied:

Theorem 2. [2] *The quotient space of complex projective planes is a* 4*D sphere, which is described by the intersection of two algebraic equations of the form*

$$a^2x - 2\,abc + b^2y + c^2z - xyz = 0, \quad x + y + z - 1 = 0.$$

As result the complex projective space $CP(2)$ in the coordinates: $z_1 = x + I \times a$, $z_2 = y + I \times b$, $z_3 = z + I \times c$ is obtained, and in this way an algebraic 3-dim hypersurface in the standard $4D$-sphere arises.

Theorem 3. [3] *The system of equations*

$$\Phi_{xy} = A^{1}_{12}\Phi_x + A^{2}_{12}\Phi_y + A_{12}\,\Phi, \quad \Phi_{xz} = A^{1}_{13}\Phi_x + A^{3}_{13}\Phi_z + A_{13}\,\Phi,$$

$$\Phi_{yz} = A^{2}_{23}\Phi_y + A^{3}_{22}\Phi_y + A_{23}\Phi$$

with coefficients $A^i_{jk} = A^i_{jk}(x, y, z)$ defines a triple conjugate system of lines in a 4-dimensional projective space, whose invariants coincide with Liouville-Tresse-Cartan invariants of equation (1).

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LAPLACE-BELTRAMI EQUATION ON LIPSCHITZ HYPERSURFACES IN THE GENERIC BESSEL POTENTIAL SPACES

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The purpose of the presentation is to expose a new approach to the investigation of boundary value problems (BVPs) for the Laplace-Beltrami equation on a hypersurface $\mathcal{S} \subset \mathbb{R}^3$ with the Lipschitz boundary $\Gamma = \partial\mathcal{S}$, containing a finite number of angular points (knots) c_j of magnitude α_j, $j = 1, 2, \ldots, n$. The Dirichlet, Neumann and mixed type BVPs are considered in a non-classical setting, when solutions are sought in the generic Bessel potential spaces (GBPS) $\mathbb{GH}^s_p(\mathcal{S}, \rho)$, $s > 1/p$, $1 < p < \infty$ with weight $\rho(t) = \prod\limits_{j=1}^{n} |t - c_j|^{\gamma_j}$. By the localization the problem is reduced to the investigation of Model Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domain $\Omega_{\alpha_j} \subset \mathbb{R}^2$ of magnitude α_j, $j = 1, 2 \ldots, n$. Further the model problem in the GBPS with weight $\mathbb{GH}^s_p(\Omega_{\alpha_j}, t^{\gamma_j})$ is investigated by means of Mellin convolution operators on the semi-axes $\mathbb{R}^+ = (0, \infty)$. Explicit criteria for the Fredholm property and the unique solvability of the initial BVPs are obtained and singularities of solutions at knots to the mentioned BVPs are indicated. In contrast to the same BVPs in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{S})$, the Fredholm property in the GBPS $\mathbb{GH}^s_p(\mathcal{S}, \rho)$ with weight is independent of the smoothness parameter s.

SHADOWING IN A PROJECTIVE IFS WITH CONDENSATION

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The Shadowing theory begins in the middle of 70th in the works of D. Anosov and R. Bowen as a powerful tool for studying the behaviour of diffeomorphisms near hyperbolic sets. Later on this concept became itself an object in its own rights with various applications, mainly for detecting chaos in concrete dynamical systems. In [1] a generalization of the notion of Shadowing to set-valued dynamics has been proposed, including those, generated by Iterated Function System's (IFS, for short) and by iterations of multi-functions or by closed relations in topological spaces. It was proved that (weakly) contracting relations have the Shadowing Property and an example of a non-contracting IFS, without the Shadowing Property, has been proposed.

In [2] it has been proved, that a scalar affine IFS has the Shadowing Property if and only if it is contracting or (strictly) expanding. If this IFS is endowed with a condensation, i.e. a constant compact-valued function is added, then the new set-valued function, named also as an IFS with condensation, has the Shadowing property if and only if it is contractive [2]. In [3] the authors prove the limit shadowing property for a gradient-like diffeomorphism of the circle with two non degenerated fixed points, and a constant function with value different from the repeller.

Our goal is to establish various Shadowing properties for projective IFS, as well as in the presence of the condensation phenomenon.

The behaviour of general projective IFS seems to be quite complicated. Even for Möbius IFS on the Riemann's sphere there is no condition to guaranty that the IFS has an attractor [4], and, in this way to prove that such an IFS admits some kind of shadowing. In [5] the authors prove that a real projective IFS has at most one attractor, and they put in evidence a class of projective IFS's that have attractor. Applying a general result from [1] and the criteria of the existence of the attractor from [5], we establish some sufficient conditions on a projective IFS to admit the Shadowing property, including those, which are endowed with a condensation.

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ON KB-OPERATORS ON BANACH LATTICES

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A complete normed lattice is called a Banach lattice. An operator $T : E \to X$ from a Banach lattice E into a Banach space X is called a KB-operator if for every positive increasing sequence (x_n) in the closed unit ball of E, (Tx_n) converges in X. An operator $T : E \to E$ on a Banach lattice E is called unbounded demi KB-operator if for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is unbounded norm convergent to $x \in E$, there is an unbounded norm convergent subsequence of (x_n).

In this presentation, we study the unbounded demi KB-operators on a Banach lattice.

ON FRACTALS GENERATED BY A TYPE OF IFSs WITH CONDENSATION

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Constructing plane fractals using computer simulations is an interesting problem. We show how this method can be applied to construct plane fractals, generated by a type of Iterated Function Systems with condensation.

Consider the Euclidean space R^n. A *hyperbolic Iterated Function System* (*IFS*) $\{\mathrm{R}^n; f_1, \ldots, f_m\}$ is defined as a finite collection of pairwise distinct contractions $f_i : \mathrm{R}^n \to \mathrm{R}^n$ $(1 \leq i \leq m)$.

M. Barnsley (1988) has introduced the concept of Iterated Function System with condensation. A constant compact-valued function f_0 on R^n with $f_0(x) \equiv K$ for some compact subset $K \subset \mathrm{R}^n$ and any $x \in \mathrm{R}^n$, is called a *condensation* with K as *condensation set.* A hyperbolic *Iterated Function System with condensation* $\{\mathrm{R}^n; f_0, f_1, \ldots, f_m\}$ consists of a condensation f_0 and contractions $f_1, \ldots, f_m$ on R^n.

It is known that a nonempty compact set $A \subset \mathrm{R}^n$ is the attractor of a hyperbolic IFS (with condensation) iff it is the unique fixed point of the respective Barnsley-Hutchinson operator on the set of compact subsets of R^n.

J. E. Hutchinson (1980) has proved that any hyperbolic IFS in a complete metric space possesses a compact attractor. M. Barnsley (1988) has proved that any hyperbolic IFS with condensation also has a compact attractor.

We have shown [1] that any convex compact set in R^n can be represented as the attractor of a hyperbolic IFS. This provides an opportunity to replace any hyperbolic IFS with condensation of a special type with a standard hyperbolic IFS, having the same attractor.

Theorem 1. [1] *Given a finite family of convex compacta $\{K_1, \ldots, K_m\}$ in R^n, let $\{\mathrm{R}^n; \varphi_{i1}, \ldots, \varphi_{ik_i}\}$ stand for the corresponding hyperbolic IFS, having K_i as attractor.*

Then $K = \bigcup_{i=1}^m K_i$ is the attractor of the hyperbolic IFS $\{\mathrm{R}^n; \psi_{11}, \ldots, \psi_{1k_1}, \ldots, \psi_{m1}, \ldots, \psi_{mk_m}\}$, where $\psi_{ij} = \mathrm{Pr}_i \circ \varphi_{ij}$ and Pr_i is the metric projection onto K_i.

Theorem 2. *Assume that:*

1. The compact set $K \subset \mathrm{R}^n$ is a union of convex compacta $K = \bigcup_{i=1}^m K_i$, where each K_i is the attractor of a hyperbolic IFS $\{\mathbb{R}^n; \varphi_{i1}, \ldots, \varphi_{ik_i}\}$;

2. $\mathrm{Pr}_i : \mathrm{R}^n \to K_i$ $(1 \leq i \leq m)$ are metric projections;

3. $\{\mathrm{R}^n; f_0, f_1, \ldots, f_r\}$ is a hyperbolic IFS with condensation, having K as the condensation set.

Then the hyperbolic IFS

$$\{\mathrm{R}^n; \psi_{11}, \ldots, \psi_{1k_1}, \ldots, \psi_{m1}, \ldots, \psi_{mk_m}, f_1, \ldots, f_r\},$$

where $\psi_{ij} = \mathrm{Pr}_i \circ \varphi_{ij}$, *has the same attractor as the given IFS with condensation.*

Applying the stochastic algorithm, we can use this result to construct new plane compact sets by computer simulations.

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REACTION-DIFFUSION SYSTEMS: BOUNDARY CONTROL PROBLEMS AND INVERSE SOURCE STABILITY ESTIMATES WITH BOUNDARY OBSERVATIONS

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We consider systems of reaction-diffusion equations in annular domains from $\mathbb{R}^d$, coupled in zero order terms and with general homogeneous mixed boundary conditions. We establish Lipschitz estimates in L^2 for the source in terms of the solution or its normal derivative on a connected component of the boundary. The main tool is an appropriate Carleman estimate in L^2-norm for nonhomogeneous parabolic systems with boundary observation.

We also discuss the use of these estimates to feedback stabilization of time varying solutions.

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STABILITY CONDITIONS OF UNPERTURBED MOTION GOVERNED BY CRITICAL THREE-DIMENSIONAL DIFFERENTIAL SYSTEM OF DARBOUX TYPE WITH NONLINEARITIES OF DEGREE FOUR

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We examine the three-dimensional differential system with nonlinearities of degree four

$$\dot{x}^j = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta} x^\alpha x^\beta x^\gamma x^\delta \quad (j, \alpha, \beta, \gamma, \delta = \overline{1, 3}), \tag{1}$$

where $a^j_{\alpha\beta\gamma,\delta}$ is a symmetric tensor in the lower indices, by which a total convolution is carried out here. By a center-affne transformation, the system (1) can be brought to the critical Lyapunov form [1] and in the center-affne condition $\eta_1 = a^\alpha_{\beta\gamma\delta} x^\beta x^\gamma x^\delta x^\mu y^\nu \varepsilon_{\alpha\mu\nu} \equiv 0$, from [2], the system (1) becomes a critical of Darboux type, of the form

$$\dot{x} = 4xR(x, y, z), \ \dot{y} = px+qy+rz+4yR(x, y, z), \ \dot{z} = sx+my+nz+4zR(x, y, z), \tag{2}$$

where $R(x, y, z) = a_1x^3 + a_2y^3 + a_3z^3 + 3a_4x^2y + 3a_5x^2z + 3a_6xy^2 + 3a_7xz^2 +$ $+3a_8xyz + 3a_9y^2z + 3a_{10}yz^2$, $a^2_1 = p, a^2_2 = q, a^2_3 = r, a^3_1 = s$, $a^3_2 = m, a^3_3 = n$, and $m, n, p, q, r, s, a_i (i = \overline{1, 10})$ are real coefficients.

According to [3], a center-affine invariant condition, which assures that the system (1) is critical, is $L_{2,3} \equiv \frac{1}{2}(\theta_1^2 - \theta_2) = nq - mr > 0$, where $\theta_1 = a^\alpha_\alpha$, $\theta_2 = a^\alpha_\beta a^\beta_\alpha$, are center-affine comitants of the system (1), from [2].

We introduce the following notations:

$$A_1 = (rs - np)L^{-1}_{2,3}, \ \ B_1 = (mp - qs)L^{-1}_{2,3}. \tag{3}$$

Taking into account the Lyapunov Theorem [1, §32] and the expressions (3), we obtain the following result.

Theorem. *The stability of the unperturbed motion, described by the critical system of Darboux type of perturbed motion* (2), *includes all possible cases in the following two*:

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I. $a_1 + a_2 A_1^3 + 3a_4 A_1 + 3a_6 A_1^2 + 3a_5 B_1 + 3a_8 A_1 B_1 + 3a_9 A_1^2 B_1 + 3a_{10} A_1 B_1^2 + 3a_7 B_1^2 + a_3 B_1^3 \neq 0$, *then the unperturbed motion is **unstable**;*

II. $a_1 + a_2 A_1^3 + 3a_4 A_1 + 3a_6 A_1^2 + 3a_5 B_1 + 3a_8 A_1 B_1 + 3a_9 A_1^2 B_1 + 3a_{10} A_1 B_1^2 + 3a_7 B_1^2 + a_3 B_1^3 = 0$, *then the unperturbed motion is **stable**.*

In the last case, the unperturbed motion belongs to some continuous series of stabilized motions, and, moreover, this motion is asymptotically stable.

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HYPERBOLIC SINGULAR PERTURBATIONS FOR ABSTRACT SEMILINEAR PARABOLIC SYSTEMS

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Let H and V be two real Hilbert spaces, such that V is continuously embedded in H. In the space H we consider the following Cauchy problem:

$$\begin{cases} \varepsilon u_\varepsilon''(t) + u_\varepsilon'(t) + A u_\varepsilon(t) + B\big(u_\varepsilon(t)\big) = f_\varepsilon(t), & t \in (0,T), \\ u_\varepsilon(0) = u_{0\varepsilon}, \quad u_\varepsilon'(0) = u_{1\varepsilon}, \end{cases} \qquad (P_\varepsilon)$$

where $A : V \subset H \to H$ is a linear self-adjoint operator and B is a nonlinear $A^{1/2}$ lipschitzian or a monotone operator, $u_{0\varepsilon}, u_{1\varepsilon} \in H, f_\varepsilon : [0,T] \to H$ and ε is a small parameter. We study the behaviour of solutions to the problem (P_ε), as $\varepsilon \to 0$, relative to the corresponding solutions to the unperturbed problem (P_0):

$$\begin{cases} v'(t) + Av(t) + B\big(v(t)\big) = f(t), & t \in (0,T), \\ v(0) = u_0. \end{cases} \qquad (P_0).$$

If $||u_{0\,\varepsilon} - u_0||_V \to 0$, $||u_{1\,\varepsilon} - u_1||_H \to 0$, $||f_\varepsilon - f||_{W^{1,2}(0,T;H)} \to 0$, using the relationship between solutions to the systems (P_ε) and (P_0) and a priori estimates of solutions to the system (P_ε) we prove that

$$u_\varepsilon \to v \quad \text{in} \quad C([0,T];H) \cap L^\infty(0,T;V), \quad \text{as} \quad \varepsilon \to 0.$$

[13] *Speaking author*: A. Perjan

It means that the perturbations of the system (P_0) by the system (P_ε) are regular in the indicated norms. At the same time, we show that

$$u'_\varepsilon - v' - \alpha_\varepsilon\, e^{-t/\varepsilon} \to 0 \quad \text{in} \quad C([0,T];H) \cap L^\infty(0,T;V) \quad \text{as} \quad \varepsilon \to 0, \tag{4}$$

where $\alpha_\varepsilon = f_\varepsilon(0) - u_{1\varepsilon} - Au_{0\varepsilon} - B(u_{0\varepsilon})$. It means that the derivatives of solutions to the problem (P_ε) do not converge to the derivatives of the corresponding solutions to problem (P_0), when $\varepsilon \to 0$. The relation (4) shows that the derivative u' has a singular behaviour relative to $\varepsilon \to 0$ in the neighborhood of $t = 0$. The singular behaviour is determined by the function $\alpha\, e^{-t/\varepsilon}$, which is *the boundary layer function* and the neighborhood of $t = 0$ is *the boundary layer* for u'.

The mathematical model (P_ε) governs various physical processes, which are described by the Klein-Gordon equation, the Sine-Gordon equation, the Plate equation, the Cahn-Hilliard equation and others equations. Therefore, we apply these results to the mentioned equations.

ABOUT METHODS FOR COMPUTING THE ORDINARY HILBERT SERIES FOR SIBIRSKY GRADED ALGEBRAS

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The Hilbert series have a special importance for some problems of qualitative theory of differential systems.

From [1] it is known a generalized Sylvester method of computing the Hilbert series for graded Sibirsky algebras of differential systems.

In the paper [2] the method of computing Hilbert series of invariants ring is described. Using Residue Theorem and the corresponding generating function [1] this method was adapted to computing an ordinary Hilbert series for Sibirsky graded algebras of comitants and invariants of differential systems [3].

Theorem. *For the differential system $s(3,5)$ the following ordinary Hilbert* *series for Sibirsky graded algebras of comitants and invariants was obtained*

$$H_{S_{3,5}}(t)=\frac{1}{(1+c)^2(1-c^2)^4(1-c^4)^3(1-c^3)^7(1-c^5)^4(1-c^7)}(1+2c+2c^2+$$
$$+8c^3+49c^4+179c^5+533c^6+1382c^7+3301c^8+7356c^9+15353c^{10}+29865c^{11}+$$
$$+54402c^{12}+93137c^{13}+150665c^{14}+231125c^{15}+337272c^{16}+468744c^{17}+$$
$$+621438c^{18}+786783c^{19}+952653c^{20}+1104296c^{21}+1226739c^{22}+1306380c^{23}+$$
$$+1334077c^{24}+1306380c^{25}+1226739c^{26}+1104296c^{27}+952653c^{28}+786783c^{29}+$$
$$+621438c^{30}+468744c^{31}+337272c^{32}+231125c^{33}+150665c^{34}+93137c^{35}+$$
$$+54402c^{36}+29865c^{37}+15353c^{38}+7356c^{39}+3301c^{40}+1382c^{41}+533c^{42}+$$
$$+179c^{43}+49c^{44}+8c^{45}+2c^{46}+2c^{47}+c^{48}),$$

$$H_{SI_{3,5}}(t)=\frac{1}{(1-c)^4(1+c)^5(1-c^4)^4(1-c^3)^6(1-c^5)^3}(1+c+c^2+7c^3+36c^4+$$
$$+106c^5+290c^6+672c^7+1451c^8+2875c^9+5322c^{10}+9053c^{11}+14398c^{12}+$$
$$+21263c^{13}+29463c^{14}+38314c^{15}+47076c^{16}+54444c^{17}+59516c^{18}+61259c^{19}+$$
$$+59516c^{20}+54444c^{21}+47076c^{22}+38314c^{23}+29463c^{24}+21263c^{25}+14398c^{26}+$$
$$+9053c^{27}+5322c^{28}+2875c^{29}+1451c^{30}+672c^{31}+290c^{32}+106c^{33}+36c^{34}+$$
$$+7c^{35}+c^{36}+c^{37}+c^{38})$$

From this theorem it follows that a Krull dimension of Sibirsky graded algebra $S_{3,5}$ ($SI_{3,5}$) is equal to 19 (17).

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CHARACTERIZATION OF THE FAMILY OF QUADRATIC DIFFERENTIAL SYSTEMS WITH AN INVARIANT PARABOLA IN TERMS OF INVARIANT POLYNOMIALS

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Consider the family of real planar polynomial differential systems

$$x' = p(x, y), \quad y' = q(x, y),$$

$p(x, y), q(x, y) \in \mathbb{R}[x, y]$, $n = \max\{\deg(p), \deg(q)\}$; $n = 2$ - quadratic systems.

Consider the class **QS** of all non-degenerate planar quadratic systems and its subclass **QSP** of all its systems possessing an invariant parabola.

A characterization of systems in **QSP** is given in [1], where the authors say that a system is in **QSP** if and only if there exists an affine transformation and a time rescaling that brings the system to the specified normal form. But this characterization involves an existential quantifier and to verify that a quadratic system is in this family we need to actually find a transformation that brings it to that specified normal form. For another system, another transformation is needed. Rigidly binding the study of **QSP** to just one normal form is not convenient. In case we study a family of systems belonging to **QSP** but presented in a different normal form, then how do we transfer results from one normal form to another? We need that our final results could be applied to *any normal form*, in other words that they must be independent of specific normal forms. This can be achieved by using the theory of invariant polynomials elaborated by C. Sibirschi [2] and developed by his disciples.

Our goal is to obtain a characterization of systems in **QSP** in terms of invariant polynomials.

We provide necessary and sufficient conditions for a system in **QS** to have at least one invariant parabola. We give the global "bifurcation" diagram of the family **QS** which indicates where a parabola is present or absent. Moreover, the diagram indicates how many parabolas exist as well as their multiplicity. Our equalities and inequalities in the bifurcation diagram splitting the parameter space into regions and subsets with distinct dynamics, will be expressed in terms of invariant polynomials which are very supple objects that can be easily

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be computed by a computer for any specific normal form and allowing us also to easily pass from one normal form to any other.

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SYSTEM DYNAMICS AS A TOOL FOR SOCIAL DEVELOPMENT PLANNING

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The evolution of a person and, with it, society, presupposes the existence of an individual opinion, which is a projection of social processes. To change the state of a global system or subsystem, it is necessary to evaluate the impact of processes on the system. One of the forecasting tools is system dynamics. Suppose the task is to maintain societal stability, that is, to achieve a consensus of opinions. Let stability be expressed by the coefficients, and society be the set of opinions of citizens (individuals) x_i. Then the general state of the system will look like this:

$$s = \begin{cases} s = 1, if \frac{1}{n} \sum\limits_{i=1}^{n} x_i \geq 0.5 \\ s = 0, if \frac{1}{n} \sum\limits_{i=1}^{n} x_i < 0.5 \end{cases} . \quad (1)$$

where x_i is the individual opinion of a citizen, in comparison with the main political doctrine of the country, which is manifested through the country's political power; i is the analysed part of the country's population ($i = \overline{1, n}$). The individual opinion of a citizen is determined by the formula:

$$f(x_i) = \int\limits_0^1 \alpha \cdot x \cdot f(p) \cdot f(c) dx. \quad (2)$$

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where α is the index of the impact of the current socio-economic situation on the state of citizens (groups, segments, etc.); $f(p)$ is the ability of a political force to influence a change in the state of citizens, including through the media; $f(c)$ is individual cognitive abilities of citizens.

It is incorrect to consider all citizens' characteristics as an average, uniform set of parameters. If we use a normal distribution, we can assume that the behavior of up to 90 percent of citizens is well predictable.

The main interest in system planning is the ability to predict the behavior of individual opinion leaders who create trends in society and the impact on the state of the system of their decisions. Thus, to achieve the desired state of the system (1), it is necessary to focus on three parameters (α, $f(p)$, $f(c)$) for conditional opinion leaders. They may have an opinion that does not correspond to the development strategy of society and acts as the main driving force capable of influencing 90 percent of the country's population, both for positive changes in the state of the system and contributing to its deterioration.

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BOOLEAN ASYNCHRONOUS SYSTEMS: REVISITING COMMUTATIVITY

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The digital circuits (i.e. the boolean systems) have been widely studied informally in electronics. Asynchronicity represents that general case in this study when each computation is made independently on the other computations. And commutativity is the weakest request that we have found, defined in [1] as: 'if two transitions can occur in either order from a given state, then there is at least one common resulting state which is independent of the order'. We identify the boolean asynchronous systems with the functions $\Phi : \{0,1\}^n \to \{0,1\}^n$ and our main concern is to formalize the concept of commutative system, using the framework from [2]. Some conclusions are presented.

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III. APPLIED MATHEMATICS

ON POLYNOMIAL AND MATRICES ASSOCIATED WITH CONNECTED NEIGHBORHOOD SYSTEMS OF CONVEX SETS

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In this paper, we introduced the concept of convex connected neighborhood polynomials and convex connected neighborhood matrices of graphs and established the convex connected neighborhood polynomials of some special graphs and graphs resulting from some binary graph operations. Moreover, we established the ranks and eigenvalues of the matrices associated with the connected neighborhood systems of the convex subgraphs of a given graph.

OPTIMIZATION OF PRODUCTION AND TRANSPORTATION PROCESSES WITH RESTRICTIONS ON ENVIRONMENTAL POLLUTION

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The research is focuses on a production system management problem where the primary objective is the maximization of expected profit. The analyzed model takes into account the optimization of production processes in the aspect of efficient use of available resources, while also including the costs associated with transporting resources from suppliers and goods to consumers. The model also considers demand and supply of the goods to be produced. This means that the amount of goods demanded by the market is taken into account, and

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[17] *Speaking author*: A. Baractari

production is planned based on demand and the production system's capacity. In addition, considering that it is important to assess the harmful impact of production and transport activities, the model includes several restrictions aimed at the harmful emissions generated by these activities, in an attempt to minimize their impact on the environment. In the objective function we could also take into account some numerical effects in the form of penalties. These being borne by the manufacturer in case of ignoring or violating some or all restrictions regarding environmental pollution.

Overall, this model offers a holistic approach to production system management, considering multiple variables and restrictions to maximize profit and minimize environmental impact. Therefore, it is useful in planning and making decisions in the production and transportation systems in a more efficient and sustainable way.

The further development of the models analyzed in the paper is to include new dynamic aspects, which could be used to assess the impact of public policy or technology changes on the environment, thus contributing to the development of effective and cost-effective strategies for reducing pollution and increasing the efficiency of processes of environmental management.

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RELIABILITY INDICATORS OF MARKOV AND SEMI-MARKOV SYSTEMS: MODELLING AND ESTIMATION

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The purpose of our talk is twofold: to investigate reliability theory of Markov systems, on the one hand, and also of semi-Markov systems, on the other hand.

We start by deriving reliability indicators of systems modelled by Markov processes; we also address associated statistical estimation questions. Then, we continue by briefly introducing the semi-Markov framework and by giving some basic definitions and results. These results are applied in order to obtain closed forms for some survival or reliability indicators, like survival/reliability function, availability, mean hitting times, etc. The last part of our talk is devoted to the estimation of the main characteristics of a semi-Markov system (semi-Markov kernel, semi-Markov transition probabilities, etc.), to the asymptotic properties of these estimators and to the estimation of the reliability indicators. Our talk is mainly based on [1-2] and [5-6].

Additionally, depending on the available time, I can also present some particular problems that are of interest in reliability theory. This part is mainly based on [3-4].

Parts of this talk are joint works with C. Ayhar, F. Mokhtari, S. Rahmani, University of Saida-Doctor Moulay Taher, Algeria; G. d'Amico, University G. d'Annunzio of Chieti-Pescara, Italy; Th. Gkelsinis, University of Rouen-Normandy; A. Karagrigoriou, University of the Aegean, Greece; N. Limnios, Université de Technologie de Compiègne, France; A. Makrides, University of Cyprus, Cyprus.

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MODELING OF NDT ELECTROMAGNETIC FOR CONCRETE STRUCTURES

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In the field of concrete structures, it is known that the controlled structure has never been tested without leaving an eventual damage on this structure.

Material sampling, due to removing cores from a concrete structure, is a destructive practice that can decline the structure performance and affects its durability.

Therefore, it is important to develop NDT techniques which allow to examine the structure without damaging it.

From all the existing methods, NDT electromagnetic (NDT-EM) techniques are frequently used in order to determine the existence or none of damages or discontinuities that may have an effect on the future usefulness of the tested concrete structure. The NDT-EM examination has become a greater control tool for assessing the condition of in-situ concrete structures.

The objective of this study is to present the NDT-EM techniques for detection and evaluation of concrete structures. The inspection by NDT-EM techniques consists to study the properties of the electromagnetic waves and their interaction with the tested structure. Thus, it's to report the physical and mathematical bases of this interaction.

The propagation of electromagnetic waves is governed by Helmholtz equations which are derived from Maxwell's equations. The study of the effect of this propagation depends primarily on the requested frequency and parameters on controlled material (conductivity and permeability). It is mainly governed by the dielectric properties of this material.

The finite element modeling, used in this study, enables to evaluate the response of a concrete dam structure to the presence of an electromagnetic wave in order to determine the characteristic frequency which presents the limit between the various modes and phenomena to be considered such as conduction and polarization.

The used approach of modeling presents a significant adequacy for the various parameters determination in particular the conductivity and the resistivity generally used for the establishment of deferent electromagnetic methods of non destructive testing.

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The obtained values of permittivity and conductivity, for different concrete dam contents, are in very good agreement to those given by above scientific and technical papers.

MODELING OF EDDY CURRENTS PROBLEM USING THE SHELLS ELEMENTS METHOD

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The modeling of the Non-Destructive Testing (NDT) by Eddy Currents (EC) problem has a vital interest, in particular in the field of aeronautics. This modeling can be used to obtain simulations that make possible to improve the test performances and to characterize defects and various structures including the structures whose thickness are low.

The finite element method (MEF) is one of the methods most adapted to problems which have complex geometries of low thickness.

However, a great difficulty can be shown during the domain discretization (mesh generation). A bad mesh grid can lead to an erroneous solution.

There are several methods which can overcome this difficulty. The method of the shells elements recognized by their strong capacity of adaptation to thin geometries is used in this study.

The principal objective of this study is the programming of a code which enables to simulate thin structures which can raise difficulties of the modeling of NDT-EC problems.

The developed code is based on a numerical resolution of the Maxwell's equations using the shells elements method.

The resolution of the obtained system allows the calculation of various fields in the different zones of the considered structure, thus well that the probe impedance. The obtained results are compared with the analytical results. A good accordance between results is observed which affirms that the shells elements represent a viable solution for this type of electromagnetic modeling.

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EXACT APPROACHES FOR SOLVING THE CONVEX COVERING PROBLEM OF GRAPHS

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We study the convex covering problem of graphs and some variations of it. The general convex covering problem of graphs is defined as follows. Given a graph $G = (V, E)$ and a natural number $p \geq 2$, the objective is to determine whether there exists a convex p-cover of G. A family of sets $\boldsymbol{\mathcal{P}}(G)$ is called a convex p-cover of G if $|\boldsymbol{\mathcal{P}}(G)| = p$, every set of family $\boldsymbol{\mathcal{P}}(G)$ is convex in G, $V = \bigcup_{S \in \boldsymbol{\mathcal{P}}(G)} S$ and $S \not\subseteq \bigcup_{C \in \boldsymbol{\mathcal{P}}(G), C \neq S} C$ for every $S \in \boldsymbol{\mathcal{P}}(G)$ [1]. Here, a set $S \subseteq V$ is considered to be convex in G if, for any two $u, v \in S$, all vertices of every shortest path between u and v are in S.

The above-mentioned problems are proved to be NP-complete [2]. An enumerative approach for finding the exact solution of such problems is usually used. In this sense, we develop new exact approaches, among which the most important are recursive decomposition and binary linear programming. The both approaches are detailed described in [3]. It is clear that these approaches considerably facilitate the use of convex covers of graphs in solving different real-world problems.

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CLOUD HIGH-PERFORMANCE COMPUTING AT MSU

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There are many High-Performance Computing (HPC) trends in 2023 and one of them is Cloud Computing. Recently, at MSU was developed a cloud distributed computing infrastructure and installed a specialized software platform for open science support, including complex applications that use high-performance Cloud infrastructures and parallel multiprocessor cluster systems.

The Cloud service is oriented on variety types of jobs and is suitable for the support of open science using multiprocessor clusters of the MSU, RENAM and the Institute of Mathematics and Informatics.

Consequently, the existing Moldova State University HPC Cluster was expanded and at the moment students and researchers can access the last generation HPC Cluster at the address http://hpc2.usm.md/. The new cluster offers users 100 processors (2.6 GHz) and 272 GB RAM memory for the execution of parallel calculations, programming environments such as OpenMPI, OpenMP, MPI, C, C++, F77, F90, Python, NumPy, ScaLAPACK, PETSc and others.

Also, courses and guides for using the cluster have been developed and can be accessed on the web address http://hpc2.usm.md/. A free online course were published regarding parallel programming, so the students will study the parallel software package ScaLAPACK, and use the libraries in developing and launching their application on the cluster.

In summary, the HPC sector in the Republic of Moldova is constantly improving and expanding, and the MSU HPC Cloud Cluster brings new opportunities for students, teachers and researchers.

ALGORITHM FOR APPROXIMATING THE SOLUTION OF THE FREDHOLM INTEGRAL EQUATION WITH DISCONTINUOUS RIGHT-HAND SIDE

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We consider the problem of approximating the solution of Fredholm linear integral equation of the second-kind $\varphi(t) - \lambda \int_\Gamma K(t,s)\varphi(s)\,ds = f(t)\,,\ t \in \Gamma$, where the right-hand side f is a function that admits a finite number of jump discontinuities on Γ. Here, Γ represents a closed interval on the real axis or a simple closed contour in the complex plane. For the equation defined on the interval $\Gamma = [a,b]$, an algorithm has been proposed in [1-2] that determines a sequence of approximations converging to the exact solution in the norm of the Lebesgue space L_p $(p \geq 2)$. But for applications, it is necessary to construct approximations that converge pointwise to the solution φ.

It is known that approximations based on the use of algebraic polynomials or standard spline functions of order $m \geq 2$ do not converge pointwise, as they

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exhibit large oscillations near points of discontinuity. When the solution of the integral equation is approximated using a combination of linear splines, the convergence of these approximations can be extremely slow [3]. Additionally, when Γ is a contour in the complex plane, the appearance of the approximations may be distorted, considering that continuous linear curves are constructed to approximate a discontinuous curve. The case when the piecewise continuous functions f and φ are defined on the interval $[a, b]$ of the real axis has been examined in [3], where it was shown that the oscillatory effect near points of discontinuity can be eliminated if the approximation is constructed as a linear combination of B-spline functions.

We propose an efficient approximation scheme for the solution of the integral equation when the coefficients f, K and the solution φ are defined on a simple closed and piecewise smooth contour in the complex plane. Additionally, the right-hand side f is numerically defined on the set of points belonging to the contour Γ. According to the proposed algorithm, the piecewise continuous solution φ of the integral equation is approximated by a linear combination of B-spline functions and discontinuous Heaviside functions defined on Γ. Theoretical and practical aspects of the convergence of the algorithm are presented, including the vicinity of the discontinuity points [4].

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RELIABILITY ANALYSIS OF SEMI-MARKOVIAN SYSTEMS WITH REBUILD STRUCTURE

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A method of reliability analysis of complex semi-markovian system with rebuild structure is suggested. It is based on modelling of their evolution by means of semi-Markov processes [1]. The results are obtained under general assumptions concerning the distribution of repair and failure times of the system units.

In should be mentioned that the results are obtained in terms of structure and means of failure and repair times and in a suitable for coding form.

The results were obtained within the Institutional Research Project 20.80009. 5007.26 concluded with the Ministry of Education and Research of the Republic of Moldova.

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THE GENERALIZATION OF EXPRESSIBILITY IN INTERMEDIATES LOGICS

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The intermediates logics are constructed on finit or infinit chains (i.e. linear ordered sets) of the values. It is well-known that a logic is called a chain if the formula $((p \supset q) \vee (p \supset q))$ is true.

The funcition f of the algebra A is called parametrically expressed by means of a system of functions Σ if there exist functions $g_1, h_1, \dots, g_r, h_r$ which are expressed explicitly via the system Σ using superposition such that the predicate $f(x_1, \dots, x_n) = x_{n+1}$ is equivalent to the predicate $\exists\, t_1 \exists\, t_2 \dots \exists\, t_l \big((g_1 = h_1) \& \dots \& (g_r = h_r)\big)$ on A. If $t_1, t_2, \dots, t_l$ are absent, then it is called implicit expressibility. The function f is called implicitly reducible to system Σ if there exists such sequence of functions $f_1, f_2, \dots, f_m$ that $f_m = f$ and f_i is implicitly expressible via $\Sigma \cup \{f_1, f_2, \dots, f_{i-1}\}$, for each $i = 1, 2, \dots, m$.

Let $Z_m = \big\langle \{0, r_1, r_2, \dots, r_{m-2}, 1\}; \Omega \big\rangle$ be a pseudo-Boolean algebra, where $0 < r_1 < r_2 < \dots < r_{m-2} < 1$, $\Omega = \{\&, \vee, \supset, \neg\}$ and LZ_m denotes the set of valid formulas, i.e. the logic of Z_m. Also, let $\varphi(0) = 0$, $\varphi(r_1) = \varphi(r_2) = r_2$, $\varphi(1) = 1$ and $\psi(0) = 0$, $\psi(r_1) = 1$, $\psi(r_2) = r_2, \psi(1) = 1$ on Z_4.

The set of all formulas in the interpretation on Z_m are permutable with the fraction f is called the formula centralizer on the algebra Z_m of function f (denoted by $\langle f \rangle$).

The system Σ of formulas is called complete by the implicit reducibility in logic LZ_m if each formula is implicitly reducible in LZ_m to Σ.

The criterion of completeness relative to implicit reducibility in LZ_3 has been obtained earlier by the author [1].

Theorem. *For any $m = 4, 5, \ldots$ in order that the system Σ of formulas could be complete by the implicit reducibility in intermediate logic LZ_m it is necessary and sufficient that Σ be complete by the implicit reducibility in intermediate logic LZ_3 and be not included in the following two formulas centralizers in algebra Z_4: $\langle \varphi(p) \rangle, \langle \psi(p) \rangle$.*

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APPROACHES FOR SOLVING BIMATRIX INFORMATIONAL EXTENDED GAME

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The bimatrix game of imperfect information on the set of informational extended strategies generates the normal form incomplete information game

$$\widetilde{\mathbf{\Gamma}} = \left\langle \{1,2\}, I, J, \left\{ AB(\alpha,\beta) = \left\| \left(a_{ij}^{\alpha\beta}, b_{ij}^{\alpha\beta} \right) \right\|_{i\in I}^{j\in J} \right\}_{a=\overline{1,t}}^{\beta=\overline{1.k}} \right\rangle.$$

We can construct the bimatrix Bayesian game for the game $\widetilde{\mathbf{\Gamma}}$ that consists of the following:(1) set of players $\{1,2\}$; (2) a set of possible actions for each player: for player 1 is $I = \{1, 2, .., n\}$, the line index, and for player 2 is $J = \{1, 2, .., m\}$, the column index; (3) a set of possible types of the player 1 are $\Delta_1 = \{\alpha = 1, ..., t\}$ and of the player 2 are $\Delta_2 = \{\beta = 1, ..., k\}$,were only player 1 (player 2) knows his type α (type β) when play begins; (4) a probability function that specifies, for each possible type of each player, a probability distribution over the other player's possible types, describing what each type of each player would believe about the other players' types $p : \Delta_1 \to \Omega(\Delta_2)$, $q : \Delta_2 \to \Omega(\Delta_1)$,where $\Omega(\Delta_2)$ (respectively $\Omega(\Delta_1)$) denotes the set of all probability distributions on a set Δ_1 (respectively Δ_2); (5) combining actions and types for each player we construct the strategies $\tilde{\imath} = i_1 i_2 ... i_\beta ... i_k \in \widetilde{I}(\alpha)$ of the player 1 ($\widetilde{j} = j_1 j_2 ... j_\alpha ... j_t \in \widetilde{J}(\beta)$ of the player 2) and it has the following meaning: the player 1 will chose the line $i_1 \in I$ from the utility matrix $A(\alpha, 1)$ if $\beta = 1$, the line $i_2 \in I$ from the utility matrix $A(\alpha, 2)$ if $\beta = 2$ and so on, line $i_k \in I$ from the utility matrix $A(\alpha, k)$ if $\beta = k$ (the player 2 will chose column $j_1 \in J$ from utility matrix $B(1,\beta)$ if $\alpha = 1$, column $j_2 \in J$ from utility matrix $B(2,\beta)$ if $\alpha = 2$ and so on, column $j_t \in J$ from utility matrix $B(t,\beta)$ if $\alpha = t$; (6) a payoff function specifies each player's expected payoff matrices for every possible combination of all player's actions and types, if the player 1 of type α chooses the pure strategy $\widetilde{i} \in \widetilde{I}(\alpha)$, and the player 2 plays some strategy $\widetilde{j} \in \widetilde{J}(\beta)$ for all $\beta \in \Delta_2$, then expected payoffs of player 1 is the following matrix $\mathbf{A}(a) = \left\|\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}} \in \widetilde{\mathbf{I}}(a)}^{\widetilde{\mathbf{j}} \in \widetilde{\mathbf{J}}(\beta)}$ where $\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}} = \sum_{\beta \in \Delta_2} p(\beta/a) a_{ij}^{\alpha\beta}$(similarly, if player 2 of type β chooses the pure strategy $\widetilde{j} \in \widetilde{J}(\beta)$ and the player 1 plays some strategy $\widetilde{i} \in \widetilde{I}(\alpha)$ for all $\alpha \in \Delta_1$, then expected payoffs of player 2 of type β is $\mathbf{B}(\beta) = \left\|\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}} \in \widetilde{\mathbf{I}}(a)}^{\widetilde{\mathbf{j}} \in \widetilde{\mathbf{J}}(\beta)}$ where $\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}} = \sum_{a \in \Delta_1} q(\alpha/\beta) b_{ij}^{\alpha\beta}$.

So for the game $\widetilde{\mathbf{\Gamma}}$ the game $\Gamma_{Bayes} = \left\langle \{1,2\}, \widetilde{\mathbf{I}}, \widetilde{\mathbf{J}}, \mathcal{A}, \mathcal{B} \right\rangle$, where $\widetilde{\mathbf{I}} = \bigcup_{\alpha \in \Delta_1} \widetilde{I}(\alpha)$, $\widetilde{\mathbf{J}} = \bigcup_{\beta \in \Delta 2} \widetilde{J}(\beta)$ and the utility matrices are $\mathcal{A} = \|\mathbf{A}(a)\|_{\alpha \in \Delta_1}$ and $\mathcal{B} = \|\mathbf{B}(\beta)\|_{\beta \in \Delta_2}$, is called the associated Bayesian game in the non informational extended strategies.

Using given above constructions and the Harsanyi theorem we get the following result.

Theorem. *The strategy profile* $(\mathbf{i}^*, \mathbf{j}^*)$ *is a Bayes-Nash equilibrium in the game* Γ_{Bayes} *if and only if, for all* $\alpha \in \Delta_1, \beta \in \Delta_2$, *the strategy profile* $(\mathbf{i}^*, \mathbf{j}^*)$ *is a Nash equilibrium for the subgame*

$$sub\Gamma_{Bayes} = \left\langle \{1,2\}, \widetilde{\mathbf{I}}(\alpha), \widetilde{\mathbf{J}}(\beta), \mathbf{A}(\alpha), \mathbf{B}(\beta) \right\rangle.$$

ASYMPTOTIC BEHAVIOR OF MARKOV PROCESSES

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Let L be a Markov process with set of states $\{x_1, x_2, \ldots, x_n\}$ and transition matrix $P = (p_{ij})_{i,j=\overline{1,n}}$, where p_{ij} is the probability of transition from x_i to x_j, $i,j = \overline{1,n}$. The matrix $P^t = (p_{ij}(t))_{i,j=\overline{1,n}}$ is the t-step transition matrix.

Let q_{ij} be the probability with which the process L occups the state x_j after a large number of transitions when the initial state is x_i, $i,j = \overline{1,n}$. The matrix $Q = (q_{ij})_{i,j=\overline{1,n}}$ represents the limit matrix and it was studied in [5]. The developed algorithm was optimized in [3], obtaining the complexity $O(n^3)$:

1. *Determination of the characteristic polynomial* $K_P(z) = |P - zI_n|$*;*
2. *Writing* $K_P(z)$ *in the form* $K_P(z) = T(z) \cdot (z-1)^{m(1)}$*, where* $T(1) \neq 0$*;*
3. *Calculation of* $Q = T(P)/S$*, where* S *is the sum of any row of* $T(P)$*.*

Next, based on results from [2] and [4], the limit and differential matrices of L are the coefficients $\overline{\beta}_k(y)$, $y \in \sigma(P)$, $k = \overline{0, m(y)-1}$, from decomposition $P(t) = \sum\limits_{y\in\sigma(P)} \sum\limits_{k=0}^{m(y)-1} t^k y^t \overline{\beta}_k(y)$, $\forall t \geq n-r$, where $\sigma(P) = \{z \in \mathbb{C} \mid K_P(z) = 0\}$, $r = deg(K_P(z))$ and $m(y)$ is the multiplicity of y, $\forall y \in \sigma(P)$. These matrices are determined in the following way, the computational complexity being $O(n^{\omega+1})$:

1. *Recursively calculate* $P^0, \ P^1, \ P^2, \ \ldots \ P^{n-1}$;
2. *Calculate* $\Pi = (p_{i,j})_{(i,j)\in\{1,\ldots,n\}\times\{1,\ldots,n\}}$*, where* $p_{i,j} = (p_{i,j}(t))_{t=n-r}^{n-1}$*;*
3. *Find* $B = (\beta_j)_{j=\overline{0,r-1}}$*, where* $\beta_t = (t^k y^t)_{(y,k)\in\sigma(P)\times\{0,\ldots,m(y)-1\}}$*,* $\forall t \geq 0$*;*
4. *Determine* $\Delta = \Pi(B^T)^{-1} = (\overline{\beta}_{i,j})_{(i,j)\in\{1,\ldots,n\}\times\{1,\ldots,n\}}$ *and build matrices.*

These algorithms were theoretically grounded in [2] and their complexity is driven by the fastest matrix multiplication [1], whose complexity is $O(n^\omega)$, where $\omega < 2.371866$. The complexity of matrix inversion [6], finding of characteristic polynomial [7] and resuming polynomials [8] is not greater than $O(n^3)$.

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AN APPLICATION OF OPTIMAL CONTROL IN QUEUEING THEORY

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We consider the $M/M/k$ queueing model modified as follows: we assume that one can decide how many servers are working at any time. Suppose that at a given time instant, there are $k+l$ customers in the system and that they are all waiting for service. Our aim is to determine how many servers should be used in order to reduce the number of customers to $k+r$, where $0 \leq r < l$, as rapidly as possible, while taking the control costs into account. The dynamic programming equation satisfied by the value function is obtained in the general case, and particular problems are solved explicitly.

COMPUTING THE STACKELBERG EQUILIBRIUM SET IN MIXED-STRATEGY $2 \times 2 \times 2$ GAME

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It is considered 3-player matrix hierarchical extended game. Each player has two strategies and a gain function. The first player is the leader for the second and three players. The second player is the successor of the first player and predecessor for the three player. The three player is the successor for the first and second players. When player j, $j = \overline{1,3}$ makes the move, he has all the information about the leaders choices, strategy sets and cost functions, but has no information about the choices of the successors players, he has all information about the strategy sets and the cost functions of the successors players. Without loss of generality, it is supposed that all players maximize the values of their cost functions. The problem of Stackelberg equilibria set determining is considered. A method for set computing is proposed, as an particular case of the method initiated earlier (see [1-2]).

The Stackelberg equilibrium set (SES) may be determined by reducing the graph of best response mapping of the third player, via a set of optimization problems, to the SES. For computing SES, the backward induction is used. According to the method described in [1], the graph of best response mapping of the third player is

$$\mathbf{Gr_3} = [0;1]^3 \cap \{X_< \times Y_< \times 0 \cup X_= \times Y_= \times [0;1] \cup X_> \times Y_> \times 1\}$$

The $\mathbf{Gr_3}$ consists of three convex components. Depending on the values of the gain matrices of third player, 59 cases are examined. As a result, 33 representations of the $\mathbf{Gr_3}$ graph are possible. The second player computes the set of his best moves on the $\mathbf{Gr_3}$. He determines the optimal values on the each non-empty component, simultaneously comparing them with the preceding value, and the best one is saved. For the components of graph $\mathbf{Gr_3}$ on faces $z = 0$ and $z = 1$ of the cube with side equal to one, in the process of determining the best response mapping of the second player, there are 9 possible cases. For component where $z \in [0; 1]$, in $x = \alpha, \alpha \in [0; 1]$, 15 cases are possible, and for $y = \beta, \beta \in (0; 1)$, likewise, the are 9 possible cases. The first player computes the set of Stackelberg equilibria, which is the set of his best moves on $\mathbf{Gr_2}$. He determines the corresponding values on the each component, he compares them and saves the best. The points of the components on which the record is attained form the SES.

Theorem. *The SES is non-empty.*

In this paper, the focus is on the possible ways of representing the graph of player three and the actions realized to determine the graph $\mathbf{Gr_2}$ (consist by at most 5 components).

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OPTIMAL STATIONARY STRATEGIES FOR STOCHASTIC POSITIONAL GAMES

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We formulate and study a class of stochastic games by applying the concept of positional games to finite state space Markov decision processes with average and discounted reward optimization criteria. We consider Markov decision processes that may be controlled by several actors (players) as follows. The set of states of the system in a Markov process is divided into several disjoint subsets that represent the position sets for the corresponding players. Each player controls the process only in his position set via the feasible actions in the corresponding states. The aim of each player is to determine which action should be taken in each state of his position set in order to maximize his own average or discounted sum of stage rewards. The step rewards in the states with respect to each player are known for an arbitrary feasible action in the corresponding states of the position sets. We consider the infinite horizon stochastic games and assume that players use stationary strategies of a selection of the actions in the states, i.e. each player in his arbitrary position uses the same action for an arbitrary discrete moment of time. For the considered class of games we are seeking for a Nash equilibrium. We show that for an arbitrary stochastic positional game with discounted payoffs there exits a stationary Nash equilibrium in pure strategies and for an arbitrary stochastic positional game with average payoffs there exists a stationary Nash equilibrium in mixed strategies. Based on constructive prove of these results we propose an approach for determining stationary Nash equilibria for the considered class of stochastic positional games. Some of these results can be found in [1-3].

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THE EVOLUTION OF THE OMICROM VIRUS AND THE INFLUENZA VIRUS IN MOLDOVA

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In this paper dynamic model of the Omicron virus and the Influenza virus spread dynamic will be examined. The dynamic system of Omicron and Influenza spread with optimal control will be formulated, aimed to minimize the number of infected population. Control function's objective consists in minimizing of the infected population and rising effectiveness of the applied instruments. Pontryagin Minimum Principle will be applied to the dynamic modelling of Omicron and influenza to find optimal solution aimed to minimize the number of infected population and control measures.

Let's consider optimal control problem of the Omicron and Influenza viruses evolution.

$$\min \int_0^T \left[I + \frac{1}{2}(u_1)^2 + \frac{1}{2}(u_2)^2 + \frac{1}{2}(u_3)^2 \right] dt$$

supposed to

$$\dot{S} = -\beta_1(I + \beta_2 E) + \sigma Q_S - u_1 S, \dot{E} = \beta_1(I + \beta_2 E)S - v_1 E - u_2 E,$$

$$\dot{I} = v_1 E - u_3 I - \gamma_1 I - \delta_1 I, \quad \dot{R} = \gamma_1 I + \gamma_1 Q_I,$$

$$\dot{Q}_S = u_1 S - \alpha Q_S - \sigma Q_S, \quad \dot{Q}_E = u_2 E + \sigma Q_S - v2 Q_E,$$

$$\dot{Q}_I = u_2 E + \sigma Q_S - v2 Q_E, \quad \dot{I}_{Inf} = v I_(inf).$$

Initial conditioun will be $S(0) \geq 0,\ E(0) \geq 0,\ \ I(0) \geq 0,\ \ R(0) \geq 0, I_{inf}(0) \geq 0,$ $Q_S(0) \geq 0, \quad Q_I(0) \geq 0, \quad Q_E(0) \geq 0$. There are:$\dot{S},\ \dot{E},\ \ \dot{I},\ \dot{R}$ modified rates of the suspected, exposed, and infected population. And $\dot{Q}_S \ \ \dot{Q}_I, \quad \dot{Q}_R$ are the modified rates of the suspected, infected, and recuperated population in quarantine, $\dot{I}_{Inf}$ is the rate of the population infected by the influenza virus. And the optimal solutions are:

$$u_1^* = max\Big(S(\lambda_1 - \lambda_5)), 1\Big),$$

$$u_2^* = max\Big(E(\lambda_2 - \lambda_6)), 1\Big),$$

$$u_3^* = max\Big(I(\lambda_3 - \lambda_7),)1\Big).$$

AN ALGORITHM FOR THE REPRESENTATION OF THE MATRIX IN THE FORM OF A MINIMAL WIDTH BAND

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It is a quite common situation when some practical problems are reduced to solving systems of linear equations. If in these equations systems the number of variables is relatively small, then their solving is done quite efficiently and there are no problems regarding the memory of the computing system and regarding the problem speed calculation. When solving problems with a large number of unknowns these problems persist. In order to reduce the required amount of memory, the matrix of coefficients of the system of linear equations is required to be presented in a"good", convenient form, for example in triangular form, in band form, in block form, etc.

The problems of transforming a matrix to a band-type matrix of minimum width usually reduce to a problem of renumbering the vertices of a graph whose incidence matrix is obtained from the original matrix and two vertices u, v of the graph are adjacent if and only if element a_{uv} is non-zero. It is clear that this procedure uniquely matches each matrix with a graph $G = (V, E)$.

In 1976 it was demonstrated [1], that if the graph is arbitrary, then the problem of determining the minimum width of the band is NP-complete. In 1978 it was demonstrated [2], that the above-mentioned problem is also NP-complete for trees with vertex valence lower than three. Based on the above, it is clear that it would be reasonable to develop approximate algorithms for solving the problem.

Currently, several approximate algorithms are known for solving the problem of finding the minimum bandwidth, the most frequently used being the Cuthilla-McKee algorithm, published by the authors in 1969.

An algorithm of the same type is presented in this paper.

At the origin of the algorithm is the following observation: if y is a vertex already numbered, and z is an unnumbered neighbor of y, then in order to reduce the bandwidth, the line corresponding to z it will be assigned a number, which was not used and which is closer to y.

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ON NUMERICAL SOLUTION OF NONLINEAR PARABOLIC MULTICOMPONENT DIFFUSION-REACTION PROBLEMS

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This work continues our previous analysis concerning the numerical solution of the multi-component mass transfer equations(see [1-2]). The present test problems are two-dimensional, parabolic, non-linear, diffusion- reaction equations. An implicit finite difference method was used to discretize the mathematical model equations. The algorithm used to solve the non-linear system resulted for each time step is the modified Picard iteration. The numerical performances of the preconditioned conjugate gradient algorithms (BICGSTAB and GMRES) in solving the linear systems of the modified Picard iteration were analysed in detail. The numerical results obtained show good numerical performances.

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DYNAMICS OF ONE-DIMENSIONAL FLOWS IN GAS-WALL CONTACT SYSTEM IN THE PRESENCE OF THERMAL IMPACTS

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The formation of one-dimensional flows of a viscous heat-conducting gas under conjugate interaction in contact systems between gas flows and shock waves with heat-conducting solids is considered. The main characteristics of emerging non-stationary flows are studied in the case of reflection of a weak shock wave from an impenetrable stationary wall and in the problem of contact between a flow of a gas at rest and a heat-conducting wall with different initial temperatures. Difficulties in the experimental study of the field of emerging flows contributed to the development of approximate approaches and methods of mathematical modeling, which allow solving the necessary various problems of continuum mechanics and studying the processes and arising phenomena.

As a result of non-stationary gas-wall interaction, gas-dynamic and thermal processes lead to the formation of a flow with a complex internal structure, described by the system of the complete Navier-Stokes equations, successfully modelling the laws of conservation of mass, momentum and energy in a wide range of change of parameters. To determine the temperature field of gas-wall, it is necessary to solve the Navier-Stokes equations together with the thermal conductivity equation for the wall. The ensemble of the Navier-Stokes equations and the heat equation with given initial and boundary conditions forms the mathematical model, describing the evolution of the parameters of a viscous heat-conducting gas and the temperature distribution of the wall.

At moderate intensity of the incident shock wave and a small jump in the initial temperatures, the perturbations of the parameters are small and the mathematical model is linearized taking into account the values of the parameters in the initial state. The linearized solution of problems in images was obtained by the method of Laplace integral transformations under the assumption of the Prandtl number Pr=1. The solution represents an expression of the parameters as a function of a complex variable which describes the dynamics of flow and the temperature distribution of the wall. On the basis of image functions, asymptotic solutions for one-dimensional flows were obtained for small and large periods of times. Asymptotic solutions for small times allow to analyze the field structure of the flow in the initial, most intense phase of the conjugate interaction, and are a necessary and convenient test tool in the development of computational methods and algorithms for numerical experiments using the non-linear Navier-Stokes equations. The obtained analytical solutions describe the dynamics of the continuous structure of the field flow with the formation of dissipative and ideal inviscid and non-heat-conducting zones.

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THE MULTICRITERIA LINEAR-FRACTIONAL OPTIMIZATION PROBLEM IN INTEGERS

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In the paper we propose a method for solving the linear-fractional multicriteria optimization model with identical denominators in whole numbers. It should be noted that such models are in growing demand, especially from a practical application point of view [2]. The procedure for solving these types of models initially involves assigning utilities (weights) to each criterion [1], at the decision-maker discretion, after which a single-criteria fractional linear optimization model is built, which is a synthetic function of all criteria weighted. It was found that the optimal solution of the weighted model does not depend on whether the optimal values of the criteria used in the objective function are of real or integer type. This finding is very important because the decision maker has the possibility to combinatorially select the type of objective function values involved in the synthesis function. By changing the utility values, we will obtain a new optimal compromise solution to the problem. Theoretical justification of the algorithm as well as a test example, which proves its effectiveness, are brought in the proposed work.

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IV. COMPUTER SCIENCE AND IT

COLOR IMAGE PROCESSING IN HWV COLOR MODEL

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Color images are three-dimensional distributions of their color stimuli and, unlike grayscale images, have a much greater information capacity. This information advantage is naturally exploited by the human visual system: a person distinguishes tens of thousands of color shades and only a few tens of shades of gray. However, for the conscious use of this advantage in image processing, a discrete color model in perceptual coordinates, such as *hue*, *brightness*, *lightness*, *saturation* is needed, which would adequately describe colors in human-friendly attributes.

Previously we have developed a pyramidal HWV color model as a discrete, one-to-one mapping into the cubic RGB hardware model. Its discrete coordinates correspond to the smallest (w), largest (v) and intermediate (h) between them components of the (r, g, b) pixel. Coordinates have strict unambiguous definition and at the same time they quantitatively and qualitatively correlate with the commonly used perceptually-based color attributes, such as *whiteness* (lightness), *brightness* and color shade (*hue*) of color.

The main disadvantage of these coordinates is their interdependence: a change in one of them leads to a distortion in another. The aim of this work consists in deducing the proportionality relations that compensate these distortions. Specific examples show that the quality of a color image is determined by the relative distribution of the chromatic (v) and achromatic (w) components in image.

THE AUTOMATION OF SOME OF THE PROCESSES FROM THE MANAGEMENT OF A DIDACTIC DEPARTMENT

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One of the important characteristics of modernization is the tendency to increase the efficiency of processes by involving information technologies. Many fields have processes that are carried out based on established and often repetitive algorithms. The field of didactic activities is not an exception to this observation. An important link in the didactic processes at the level of educational institutions is the department (chair). Many processes are cyclical, being directly determined by the notion year of studies. During the academic year, there are several actions that happen repetitively, used for organizing the didactic process. An important aspect of the activity consists of the recording and management of different types of resources: teachers, subjects, classes. As a rule, in the activity process, some aspects are standardized, fact expressed through a series of extensive reports, required at different stages of the teaching year. At the Department level, in addition to compiling many non-standard reports, there are a series of standardized documents that are generated each academic year:

1. Record of the department's teaching activity;
2. State of functions and teaching staff;
3. Teaching staff;
4. Reporting on the accomplishment of the didactic task (two reports, one after the first semester and the 2nd one at the end of the year).

It would be much better if these reports were generated automatically, but such a process would require an Artificial Intelligence system, that would include the option of changing the knowledge data base aligned on the specific circumstances of a certain year of studies. However, a reduction in the time of preparing the reports can be obtained by using information technologies. The suggested approach is based on the possibilities offered by Microsoft Office. The possibilities offered by Microsoft Excel are well known and it is clear that attempts to speed up the report generation process are based on the use of functions and formulas in Excel. Although it is a considerable step, there must be one more extremely powerful component involved in this process, and that is the possibility to program on multiple levels of many applications that are part of Microsoft Office. At the programming level in Microsoft Office, the adapted language that is used is VBA (Visual Basic for Applications). Therefore, the use of the programming mechanism takes the problem of generating reports to a whole new level.

A THEORETICAL TREATMENT OF PROGRESSIVE TAXATION

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The article considers approaches to the interpretation of the concept of fiscal equity. The possibilities of substantiating the application of a progressive tax grid with the help of diminishing marginal utility are considered. The author's understanding of the essence of a progressive tax system is presented. The possibility of using the theory of preference and the attitude towards risk of people as a tool for determining the maximum expected utility in taxation is determined. An assessment is made of the compliance of tax withdrawals to the budget with the theoretical and methodological requirements and practices of its implementation in order to increase the effectiveness of its fiscal control. In our opinion, the assessment of the uniformity of taxation should be based on the answer to the question of how comparable are the losses incurred by payers with different levels of income from paying taxes to the budget. Obviously, assessing the losses in absolute terms does not serve the purpose of ensuring a uniform distribution of the tax burden, since in the case of paying a fixed amount of tax, payers with different income levels donate a different share of their funds, in addition to the higher the income level, the lower its share will be transferred to the budget. It is therefore inappropriate to set a lump sum tax.

SPEECH PROCESSING SYSTEM FOR EMERGENCY MANAGEMENT

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Along with the automation and robotization of technological and production processes, some premises have also been created that can create exceptional situations and negatively influence the health or life of human beings. The intervention, in exceptional moments, in the functionality of the technological and production processes will essentially reduce the traumas and deaths at industrialized enterprises. The first reaction of human beings, in exceptional cases, is the generation of emotional sounds that are in a special spectral alert range. A solution would be the development of intelligent systems that will perceive sound waves. As a result of the processing of these signals, commands will be generated to pass the technological process or the production process in mode of sigurance or its stopping.

Either, the signal is defined $u(t)$ which characterizes the amplitude of sound in the space of the production process. The exceptional situation management system is made on the basis of a Neural Network with the input vector $X(t) = [x_i(t), i = \overline{1, N}]^T$ and the output vector $Y(t) = [y_j(t), j = \overline{1, M}]^T$. Matrix $W = [w_{i,j}, i = \overline{1, N}, j = \overline{1, M}]$ shows the multiplication coefficients, where: $Y(t) = WX(t)$.

Input vector values $X(t)$ are calculated based on expressions:

$x_1(t) = u(t); x_2(t) = \frac{du(t)}{dt}; x_3(t) = \frac{d^{(2)}u(t)}{dt^{(2)}}; \ldots; x_N(t) = \frac{d^{(N-1)}u(t)}{dt^{(N-1)}}$

Output vector values $Y(t)$ are calculated based on expressions:

$$\begin{cases} y_1(t) = w_{1,1}u(t) + w_{1,2}\frac{du(t)}{dt} + w_{1,3}\frac{d^{(2)}u(t)}{dt^{(2)}} + \cdots + w_{1,N}\frac{d^{(N-1)}u(t)}{dt^(N-1)}; \\ y_2(t) = w_{2,1}u(t) + w_{2,2}\frac{du(t)}{dt} + w_{2,3}\frac{d^{(2)}u(t)}{dt^{(2)}} + \cdots + w_{2,N}\frac{d^{(N-1)}u(t)}{dt^(N-1)}; \\ \ldots \\ y_m(t) = w_{m,1}u(t) + w_{m,2}\frac{du(t)}{dt} + w_{m,3}\frac{d^{(2)}u(t)}{dt^{(2)}} + \cdots + w_{m,N}\frac{d^{(N-1)}u(t)}{dt^(N-1)}. \end{cases}$$

The learning process of the Neural Network takes place by applying to its input a set of sounds that cause exceptional situations.

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SOME SCIENTIFIC AND PRACTICAL RESULTS REGARDING MENTAL AND BEHAVIORAL DISORDERS IN EPILEPSY

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The diagnosis, treatment, necessary resources, rehabilitations and prophylaxis of patients with mental and behavioral disorders in epilepsy (MBDE) are important issues for society. The authors have been studying MBDE for 20 years. The team of authors, assisted by means of artificial intelligence, researches the mechanisms of MBDE remissions in order to propose innovative methods of healing patients with MBDE. Some issues in the MBDE research field (RF) are at the frontier of knowledge. One of the authors, the psychiatrist-epileptologist Alexandru Popov, in 2003 had an experience of remission in approximately 100 patients with MBDE. Currently, doctor Alexandru Popov, assisted by the results of joint research, has achieved over 175 remissions in MBDE patients. Our research went through the following stages:

1. *Description of the MBDE research field.* RF MBDE operates with 27 MBDE diagnoses and a control diagnosis G40 (epilepsy without MBDE), 163 symptoms, associated with 17 syndromes. A book was developed [1], which includes 10 dimensions of MBDE and the MBDE Patients Support Guide. The work was appreciated with Diplomas of Excellence and Gold Medals at some International Research, Innovation and Book Fairs.

2. *Formalization of RF MBDE knowledge.* Inspired by Mendeleev's Table, the authors developed two tables of 5040 cells each. Table data is organized hierarchically in three levels. The first table integrates fuzzy numerical data. The second table – fuzzy symbolic data corresponding to the first table. Both tables integrate data on the degree of manifestation of symptoms into syndromes and diagnoses.

3. *Development of software and e-learning products for RF MBDE.*

4. *Development of 19 metric spaces for evaluating distances between MBDE RF diagnostics.*

5. *Development of 19 metric spaces for evaluating the similarities of RF MBDE diagnoses.*

An album of 76 MBDE distance and similarity evaluation tables was developed.

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manifestations, paraclinical investigations, treatment, rehabilitation, required resources, prophylaxis. CEP USM, Chisinau, 2021. (In Romanian.)

VOICE RECOGNITION SYSTEM FOR STREAMLINING DATA ENTRY IN MASS CASUALTY TRIAGE

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The importance of the triage process is self-evident in mass casualty incidents. Triage is the prioritization of patient care during any natural or man-made incident, based on injury, illness, severity, prognosis and available resources. Previously in our research we created tools for the triaging process, a mobile tool for registration and categorization purposes, it also has the E-FAST and Transportation functions [1]. In order to optimize the recording data in the first phase of the triaging process we decided to implement voice and speech recognition techniques.

This paper presents an innovative approach in the realm of emergency medical services by employing voice recognition technology within mobile applications to enhance mass casualty triage efficiency. In the face of large-scale disasters, the limitations of manual data entry in terms of time and error susceptibility pose significant challenges to effective patient care. We propose a novel solution featuring hands-free, verbal data entry by paramedics directly into a digital triage system, overcoming the hurdles associated with traditional methods. This system integrates advanced AI algorithms, enabling the interpretation based on structured verbal assessments following a defined protocol.

Voice recognition technology converts a spoken word into command or in text such as in our case. This technique has a significant impact in the healthcare field. Medical personnel normally spend a lot of time doing paperwork. It takes time to type or event to write out notes, but it is quickly to speak them aloud. In this regard, spending less time on triaging is equal to more opportunity in saving people. Thus, we developed a mobile application. The purpose of this application is to collect audio data set according to some predefined requirements. Afterwards we intend to use speech recognition techniques in order to analyze and extract the needed information and to put it into a table data base.

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To ensure accuracy and facilitate data mapping into the necessary database fields, the voice recognition system requires the use of 'keys' as prefixes before the actual values. For instance, a paramedic might say "Gender key, female," or "Heart rate key, 110." The introduction of these keys aids the system in identifying, segmenting, and correctly allocating the recognized text. This protocol-based approach significantly accelerates the data entry process, minimizes human error, and optimizes care delivery during high-pressure emergency scenarios.

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MECHANIX: SOFTWARE FOR SHAPE DRAWING AND PROGRAMMING

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In the current paper, a computer program was developed to teach the fundamentals of programming to learners interested in computer science. The strategy used is to let the learners draw their own paintings by writing algorithms. The program's environment allows the end user to import source codes, or alternatively, script files, that can be used to draw diverse shapes on a graphical container known as the canvas. In runtime, the drawing process is made possible by moving the brush, characterised by a kangaroo using its tail, around the canvas in accordance with the special commands specified in the compiled script file.

Under the hood, Mechanix consists of two proper key components: the Tian programming language and the Ciochina virtual machine. Tian is a procedural programming language used to write importable source codes, and it features variables, functions, procedures, structure types, pointers, operator overloading, etc. The keywords specified in Tian's grammar are in Romanian, effectively making Mechanix targeted to the Romanian-speaking audience. The Ciochina virtual machine acts as the bridge between Tian and the runtime. When importing a source code, it is processed for the absence of Tian-related errors (e.g., syntax errors). Afterwards, the generated syntax tree compiles into bytecode so it can be executed directly by the virtual machine at runtime.

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The scientific novelty of Mechanix is to implement functionalities that are not featured in its inspiration, Cangur. Cangur is educational software developed in 1998 [1], and is featured in a textbook for Moldovan secondary schools [2]. At the time, Cangur's internal programming language was insufficient to cover most of the programming fundamentals, as it supported only conditionals, procedures, and loops. The language used in Mechanix, Tian, is meant to solve the problem by inheriting most of the crucial language constructs featured in general-purpose programming languages such as C++, Pascal, and Python. Tian's rich grammar allows the learner to create full-blown scripts with the capability of drawing complex shapes, as opposed to ”reinventing the whee” in Cangur.

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HIERARCHICAL ITEM CODE STRUCTURE IN MOODLE

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The MOODLE learning platform has become one of the most widely used platforms, partly due to its open-source nature and continuous improvement capabilities. This paper examines an improvement to MOODLE by transitioning from an item identifier (ID), which represents a sequential number in a table, to a structured hierarchical identifier assigned in accordance with the course's hierarchical structure, item type, complexity, etc [1].

The item's order identifier in the table is a number used by the system, with low informational value for the course developer, as it does not provide any information about the item. The Moodle LMS offers users the ability to set the IDNUMBER field for a wide range of uses, such as a structured hierarchical ID. The proposed TONICO structure is flexible and can accommodate up to 6 criteria for grouping/classification, such as the item number, type and objective number, item type, complexity level, and order number. TONICO is an abbreviation for *Topic*, *Objective* (knowledge, skills/abilities, application), *Number* (of objective order), *Item type* (binary, single response, etc.), item *Complexity*, and number *Order* in this hierarchical structure. Ideally, the hierarchical

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structure of the TONICO item collections would align with the hierarchical structure of the study subject, based on a curriculum structured by topics, objectives, self-training events, self-assessments, interim and final evaluations. In such conditions, self-training and self-assessment, both formal and informal, can be easily formalized and made continuous.

Use cases:

– Facilitating the management of item collections, including automatic grouping according to the item code, importing, exporting, textual editing (Gift format), etc.

– Supporting test generation by filtering items based on predefined criteria and randomly selecting items to be included in the test.

– Enhancing transparency in post-testing analysis for item review and improvement, including easy regrouping by simply modifying the ID, reducing computational resources per item, test, backup copies, etc.

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DEVELOPMENT OF AN AUTOMATED SYSTEM FOR PROGRAMMING CONTESTS MANAGEMENT

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Currently, programming contests have become a popular means of education, self-improvement, testing, participant selection and motivation. There is a large number of different contests, among which it is worth highlighting the International Olympiad of Informatics (IOI) [1] - and the International Collegiate Programming Contest (ICPC) [2]. The National School Olympiad in Informatics serves as a qualifying round for IOI.

There are several systems available that provide organizers of programming contests [3] with tools for managing competitions and keeping track of results. Most of these systems include features such as automatic submission and solution verification, score calculation, and ranking generation. However, despite the availability of such systems, there are certain drawbacks that can hinder their usage. For instance, all international and national level competitions are automated, but until 2021, most districts in Moldova organized their competitions

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manually at the municipal level. Currently, the Romanian program Evaluator is used at the municipal level, but it only provides solution verification and comes with several issues.

One of the drawbacks of existing systems for automating the organization of programming contests is the complexity of installation and configuration. Working with these systems requires a certain level of knowledge. Additionally, most of these systems do not support the Windows operating system, which limits their usage among a wide range of organizations. Another problem is the inconvenience or even absence of importing and exporting tasks, participants, and contests. All existing systems do not provide a simple and efficient way to exchange data between different systems or file formats.

The aforementioned limitations of existing systems for automating the organization of programming contests create difficulties in organizing such contests at the municipal level. In light of these drawbacks, a new system called Olymp [4, 5, 6] is being developed to automate the organization of programming contests. The main focus is on simplifying the installation process, providing intuitive management (UX), and enhancing the capabilities of data import and export. The new system aims to provide organizers and participants with tools that will allow them to conduct programming contests more effectively and conveniently.

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MODEL OF AUTHORING SYSTEM OF COMPUTER ASSISTED COURSES GENERATOR

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Development of computer training courses is a very sophisticate laborious task. This provides the necessity of rising the authors' labour capacity by using specific design technology and tools. Automatic program generation of computer training courses is possible on the basis of this technology.

This abstract is about the theoretical concept oriented on computer-aided design and creation of computer training courses programs.

The conceptual model determines directly the software architecture, because it really represents a computer realization of the model.

Plan and program. The general task theory [1] was used for the purpose to design the conceptual model. The concept of programming problem (**ZP**) was examined and formalized on the basis of this theory. The person , as a resolving system, should have a number of operators $\mathbf{Y}= \mathbf{Q}\ \mathbf{U}\ \mathbf{F}$, where $\mathbf{Q}=\{\mathbf{q_i}\}$ is a number of planning operators, which provide the production of the plan of problem solving and $\mathbf{F}=\{\mathbf{f_j}\}$ a number of executive operators which provide program compiling. Respectively the plan is presented as a pair of $\mathbf{P}=(\{\mathbf{q_i}\},\mathbf{R_q})$, where $\mathbf{R_q}$ – is a successor relation, which sets the order of execution $\{\mathbf{q_i}\}$, and the program is represented as $\mathbf{G}=(\{\mathbf{f_j}\},\mathbf{R_f})$, where $\mathbf{R_f}$ – is a successor relation to $\{\mathbf{f_j}\}$.

Planing instructions $\{\mathbf{q_i}\}$ are not formalized and are presented on high level while the executive operators $\{\mathbf{f_j}\}$ are formalized and presented on low level in respect to the plan, because $\mathbf{f_j}$ group of **G** executive program operations corresponds to one $\mathbf{q_i}$ operation.

Thus, **ZP** includes interdependent plan **P** and program **G.** The traditional technology of solving tasks in man–machine system reflects really this situation when the person forms a variant of plan **P**, and program **G** is assembled again with the help of person by instructions of Programming Language.

The analysis and **ZP** formalization give the basis for introduction of the concept of operational model of an educational situation as a high–level tools of realization of the operators of planning $\mathbf{q_i}$ in the course script plan $\mathbf{P} = (\{\mathbf{qi}\}, \mathbf{Ri})$.

On the basis of the analysis of applied requirements of the process of the computer instruction courses creation the set of high–level operators $\mathbf{Q} = \{\mathbf{d}, \mathbf{q}, \mathbf{a}, \mathbf{r}, \mathbf{h}, \mathbf{t}, \mathbf{i}, \mathbf{b}, \mathbf{c}, \mathbf{s}, \mathbf{n}, \mathbf{f}, \mathbf{m}, \mathbf{e}\}$ of plan **P** as operational models of educational situations, where **d** – demonstration, **q** – question, **a** – answer, **r** – reaction, **h** – help, **t** – text, **i** – if, **b** – block, **c** – calculation, **s** – sound, **n** – menu, **f** – function, **m** – module, **e** – end is specified.

To the author they are known as "case–operators". Case–operators define the technological basis of modules designing from which the computer instruction course is assembled.

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COLLABORATIVE MULTI-AGENT MULTI-OBJECTIVE SYSTEM

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Today, most of the economic, technological and production processes cannot be developed in isolation, because their activity requires a close collaboration with other processes in various fields. This collaboration is carried out on the basis of Multi-Agent systems at the level of Artificial Intelligence models [1,2], which ensures: the sharing of resources and knowledge, common strategies and objectives, participatory decisions, efficient management of projects and tasks, participation and active involvement of members, etc. In these systems the collaboration process takes place on the basis of an evolutionary coalition made up of a lot of Agents $A = \{a_i, i = \overline{1,N}\}$, that meet common objectives $min/max(f(X)), \forall X \in R^N$, where $X = \{x_i, i = \overline{1,N}\}$.

The evolution of coalitions to form collaborative groups takes place on the basis of the model:

$$\begin{cases} a_i \subset A_j^{\min} \left| \frac{\partial f(x_j)}{\partial x_i} < 0, i = \overline{1,N}, j = \overline{1,K}, i \neq j \right. \\ a_i \subset A_j^{\max} \left| \frac{\partial f(x_j)}{\partial x_i} > 0, i = \overline{1,N}, j = \overline{1,K}, i \neq j \right. \end{cases} \tag{1}$$

Where: $A_j^{\min}$ - is the crowd of Agents that form the coalition j and meet the objectives $\min\big(f(X_j)\big), \forall X_j \in R^N$; $A_j^{\max}$ - is the crowd of Agents that form the coalition j and meet the objectives $\max(f(X_j)), \forall X_j \in R^N$.

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AUTOMATION OF THE SERVER-SIDE DEVELOPMENT PROCESS FOR WEB APPLICATIONS

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This research focuses on the comprehensive analysis of the server-side development process for web applications, encompassing testing and integration stages. Through this analysis, a well- defined software development scenario has been formalized. Additionally, the study examines the specific phases that can be automated, serving as the foundation for the subsequent theoretical basis of automation algorithms.

Moreover, contemporary architectural design patterns are investigated, specifically selecting patterns that adhere to the principles of automation. These selected patterns establish the fundamental templates, or stubs for the automation tool.

By conducting a comprehensive analysis of the development stages and patterns, it was possible to identify system components and processes that can be automated. The findings suggest that, given the provided specifications, a minimum of thirty percent of the code can be generated automatically.

Furthermore, this study delves into various approaches and paradigms associated with development automation. Metaprogramming and declarative programming paradigms are extensively examined, forming the basis for the meta-compiler algorithm and specification language. Consequently, precise requirements for the future compiler are formulated, the operational algorithm is formalized, and the specifications for the declaration language are derived.

Based on the aforementioned theoretical findings, a tool has been developed to automate the server-side development of web applications. As an experimental implementation, the Laravel framework has been employed. The implemented tool offers the following functionalities:

1. Processing of component specifications
2. Distribution of individual specification parts to child component compilers
3. Template population and source code generation for components.

The developed meta-compiler enables the automated creation of system components such as:

1. HTTP request descriptions
2. HTTP response descriptions
3. DTO object descriptions
4. Mapper descriptions

5. Model and migration descriptions
6. API documentation
7. Simple unit tests.

Consequently, the service layer (business logic rules) and the layer responsible for data interaction remain unaffected by these components. Therefore, the implemented meta-compiler automates the creation of eight out of the ten main components of the system, effectively streamlining the development process for the server-side of web applications.

DISTRIBUTED COORDINATION STRATEGIES IN CLOUD-EDGE CONTINUUM

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Distributed coordination represents managing inter-dependencies between positions and activities performed to achieve goals. Henry Mintzberg has defined the following mechanisms that can be applied to coordinate dependencies between positions and activities [1]: *Direct Supervision* (a central service takes all decisions by issuing instructions to agents and monitoring their actions), *Standardization of Work* (the specification of technical activity of the agent, step by step, without the possibility to negotiate, conflicts are reported to the supervisor), *Standardization of output* (agents coordinate by specifying the result), *Standardization of skills* (the specification of the competencies needed for the activity), and *Mutual Adjustment* (coordination by process of informal communication between agents). For distributed intelligent system design, such as Cloud-Edge Continuum [2], this means that agents have social abilities in the sense that they can interact and reason about each other's interfaces, knowledge and competencies, and activities to achieve, without hardly any standardization or protocols.

We proposed an architecture that has an essential layer for agent communication, management, and message transportation, as well as a specification for the abstract architecture and applications layers (see Figure 2).

[29] *Speaking author*: F. Pop

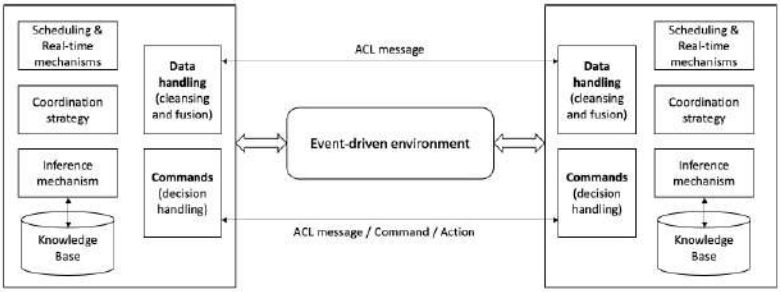

Figure 2: *Proposed architecture using coordination and scheduling strategies*

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DATA MINING FOR SOCIAL PROBLEMS

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Social systems belong to the class super complex. They include various subsystems and elements, characterized by a large number of parameters. Their education and development are determined by the interaction of various internal and external factors. Therefore, the creation of their models is always accompanied by great difficulties. From this point of view, the problem of the development of social system modeling methods based on modern methods of data analysis is relevant.

Data science is the systematic study of data with the primary purpose of making evidence-based decisions. Data mining is a key success factor for any data science analysis. At a very high level, data mining in data science involves data collection, data management, data analysis, and data reporting in support of evidence-based decisions. Insights derived from data and results of analysis form the basis for evidence to support decisions. Data science is an iterative process where one cycle of research leads to the next cycle of research to gain deeper insights and understanding based on the results of previous research.

The typical phases required in a data science process in general, and in data exploration in particular, can be listed in the following order: Identify the problem statement, Identify the problem questions; Hypothesis formulation; Data collection; Data management; Quantitative analysis; Interpretation of quantitative results and decisions; Data preparation for algorithmic model development.

We propose to make a study of the problem for selection of specializations on the part of graduates from schools. Choosing the future profession is not an easy decision, and the factors that influence this decision are numerous. It often influences both internal factors (knowledge, own preferences, skills, personal interests), but also multiple external factors (opportunities, advertising made for faculties, other people's impressions, the influence of the people around us. And of course, the most important factors are the instinct, and in many cases self-confidence. This problem can be modeled with the help of artificial intelligence and Big data.

Many of the problems associated with "Big Data" in the social sciences cannot be adequately addressed by current Big Data methodologies and technical frameworks. The reason for this is their focus on horizontal scaling and real-time computing.

CONTENT EXTRACTION FOR ELEARNING SYSTEMS

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One of the new educational technologies that has shown its undoubted effectiveness is e-learning. In developed countries, e-learning covers all levels of education and is widely used not only in universities, but also in high school and in the organization of corporate (postgraduate) education [1].

Such platforms require the elaboration of high-quality and relevant teaching resources, the constant updating of existing ones. This, in turn, is a complex process consisting of processing a variety of materials, their analysis, synthesis, creative development and processing of all elements to build a single harmonious structure [2]. Up to now, far too little attention has been paid to dynamic content generation for e-learning courses.

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In our previous work [3], we have proposed a program model for the dynamic creation of training courses and discussed the ways of content extraction and character recognition from images such as PDF, DOC, DOCX, and HTML. This paper continues our research and regards extracted text as meaningful pieces of information. Further on, using the NLP approach, we build some models to train our machine to make associations between a particular input and its corresponding output. An AI system uses statistical analysis methods to build its own "knowledge bank" and discern which features best represent the texts before making predictions for unseen data.

A constructed system of AI, by applying semantic and pragmatic analysis, selects the most common parts of information and builds some new coherent text. This text is given in the text editor for further editing and downloading.

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METHODS OF ENSURING USER PRESENCE IN VIRTUAL WORLDS

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Virtual worlds, created by people, can be treated as effective platforms for carrying out activities for humans, in different fields: in the construction of machines, in the world of academic researchers, in medicine, in tourism, in business, in the learning process, etc. The use of VR (Virtual Reality) / AR (Augmented Reality) technologies include multiple cost savings.

However, it is very important for all computer systems, which generate virtual worlds, how they can ensure the level of presence, the feeling of "being there", for a user of such a world. The efficiency of using virtual worlds has often been related to the feelings of presence reported by their users.

Currently, many processes, such as training or presentation of scenic places on Earth, are moving from 2D graphical interfaces, based on the web environment, to interactive and dynamic ones, containing virtual elements. Virtual worlds offer unique opportunities to simulate real-life scenarios. VR and AR are expected to have a big impact on teaching, knowledge and learning for generations to come.

A virtual world is a real-time computer-generated 3D virtual environment running in a virtual computer system, using either a workstation or the capabilities of the global Internet.

One of the methods users can access the virtual worlds is as simple observers, in front of the 2D screen. In this case they will only be able to follow what is happening in the virtual world, having the possibility to interact with some objects or elements through the keyboard or mouse.

Or, users can take the form of avatars, which are digital representations of people, visible to other participants in the virtual world. Avatars are controlled by users, having the ability to move freely in the virtual world, just like humans: walking, running, sitting, standing, flying or teleporting from one location to another and manipulating objects.

Or, in order to increase the level of user presence in the virtual world, immersive devices (HMD, controllers, gloves, etc.) can be used. These devices increase the perception of realism in virtual worlds, and the user feels like "being part of it". As a rule, in this case, his perceptual, cognitive/visual, auditory and motor skills are affected.

ON AI FRAMEWORKS FOR INVESTIGATION OF EXPRESSIBILITY IN LOGICAL CALCULI

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We consider problems related to functional expressibility [1] (and its variations) in a logical calculus L. Basic problems are: a) given a formula F, a

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system Σ of formulas and a set of rules R for obtaining formulas from Σ to find out weather F can be obtained from Σ by means of R, and in the case of positive answer to get out the precise way how to get it (they say F is expressible by rules R via Σ in L); b) to find out if any formula of L can be obtained from Σ by means of R (i.e. Σ is complete by rules R in L); c) to find out if Σ is almost complete relative to rules R in L, i.e. Σ in incomplete, but for any formula G, which is not expressible by rules R via Σ in L the system $\Sigma \cup \{G\}$ is already complete.

The above problems are related to complex routine calculations and are instances of search problems. The idea is to use search techniques developed in Artificial Intelligence, and especially those so called nature inspired. Related to above described problems we consider that most promised one is the technique called Genetic Programming [2].

There are different frameworks in different programming languages that provide some tools that implement concepts from Genetic Programming. For Java language we consider frameworks described in [3, 4]. For Python language we consider frameworks mentioned in [5, 6, 7].

We analyze the ease of use, the ease of prototyping programming solutions to the problems of expressibility in logical calculi using a set of defined tools. Other goal is to connect them to Jade agents [8].

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OPEN SCIENCE BASED APPROACH FOR ANALYZING PANDEMIC AND POSTPANDEMIC DATA

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The overall goal of the approach is to use basic principles of Open Science in preprocessing primary pandemic and post-pandemic data and to offer digital solutions for the healthcare domain, based on both advanced computing methods and technologies, and on mathematical modeling.

We are witnessing a widespread prevalence of liver disease associated with metabolic syndrome, which predominantly affects people of working age. It has a significant negative impact on social and economic development of the countries, and the COVID-19 pandemic only made things worse.

Metabolic Associated Fatty Liver Disease (MAFLD) was selected as a field of study and implementation of the formulated approach.

The main identified objectives are as follows:

– Ensuring the functioning of the FAIR principles for the initial pandemic and post-pandemic data set used within the project – to be Findable, Accessible, Interoperable and Reusable – in order to utilize this data further and to facilitate automated search/access;

– Development of IT tools for stratification of MAFLD patients, based on a unique formalized taxonomy and a minimum set of informative parameters, which would allow a better understanding of MAFLD influence as a risk factor in pandemic situations;

– Development of a general model for describing and visualizing the principal pandemic parameters in the cohort of patients with MAFLD comorbidity;

– Description of the “ use case ”, implementing FAIR principles in practice and promoting the Open Science values the research community.

Achieving the proposed objectives will offer a solution to preprocess existing heterogeneous primary data collections in order to fully exploit their potential [1]. Information tools for stratification and mathematical model, describing the principal pandemic parameters, will allow quantification and evaluation of the interplay of compromised liver functioning as a result of MAFLD and the severity of COVID-19 disease, as well as vice versa.

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INTELLIGENT SUPPORT SYSTEM FOR SOLVING INTEGRAL AND WEAK-SINGULAR INTEGRAL EQUATIONS OF SECOND KIND

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In this work the spline-collocations and spline-quadratures methods for solving integral equations (IE) are developed and theoretical substantiated. Also, the Intelligent Support System (ISS_IE) for approximate solving of regular and weak-singular integral Fredholm and Volterra equations of second kind is developed and implement based on the following received results:

– development of spline-collocations and spline-quadratures computational algorithms for approximate solving of regular and weak-singular integral Fredholm and Volterra equations of second kind;

– some modification of traditional quadrature-interpolations computational algorithms in solving IE. These new computational algorithms are effective and better adapted to their way of theoretical substantiation. As an example, the computational algorithms of spline-quadratures method in solving IE Fredholm and Volterra of second kind;

– theoretical substantiation in Hölder spaces and continuous functions space of developed computational algorithms;

– implementation of degenerated kernel method for exact solving the Fredholm and Volterra IE of second kind;

– establishment of principles and architecture of development ISS_IE for regular and weak-singular integral Fredholm and Volterra equations of second kind.

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Based on the provided cores for regular and weak-singular integral equations (more than 2000 types) two knowledge bases were created: * The base of prototypes of integral equation kernels (BPK_IE_COMP) for checking the sufficient compatibility conditions of integral equations; * The base of prototypes of kernels of regular and weak-singular integral equations (BPK_IE_COL) for solving by the spline-colocations method.

Integral Equations Solver (IES) for regular and weak-singular integral equations has been developed.

The obtained algorithms were tested on several examples, and was found to be quite effective.

USING AR TECHNOLOGY FOR EVALUATION

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Augmented Reality (AR) [1] has the potential to revolutionize education as a whole. With the help of AR, information is added to the environment in countless ways creating engaging content for learners, developing learning environments with unforgettable experiences, will surely increase the active involvement of students in the learning process, because when the physical and digital worlds collide, changes everything.

AR technology [2] has changed the way we interact with the real world. It is a technology that extends our physical world by adding layers of digital information on top of it. Augmented reality does not create completely artificial environments [3] to replace the real world with a virtual one, it merges the real world with a computer-generated environment. Mixing an existing environment, in real time, to which multimedia elements are added, such as: video, sounds, graphics, etc. The benefits of using AR technology in education are well known: Accessible learning materials – anytime, anywhere, no special equipment is required, higher student engagement and interest, improved collaboration capabilities, a faster and more effective learning process, practical learning, safe and efficient workplace training, universally applicable to any level of education and training. Including for the evaluation process, the possibilities offered by AR technology can be applied to realize this process interactively. To carry out the assessment the objective and subjective items were implemented. Objective

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items offer the possibility of structuring the proposed tasks, the standardization of the presentation format, the strict correlation of the tasks with the objectives to be evaluated, the ability to test a large number of content elements in a relatively short time, objectivity regarding the assessment of the answer, the possibility of associating a relatively simple grading system. Subjective items offer the possibility to assess areas of students' performance that are complex and qualitative, using questioning which may have more than one correct answer or more ways to express it. Artefacts that generate scenarios for making the following items types have been made: Multiple Choice Questions, True and False Statements, Short Questions and Answers, Essay Writing.

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THE USE OF TEMPORAL LOGIC FOR THE SYNCHRONIZATION OF DECISIONS IN THE SMART CITY

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The rapid development of embedded electronic devices, information and communication technologies, and their popularity among citizens, were the basis for the development of Smart City infrastructure [1,2]. Technological and algorithmic complexity, hardware and software heterogeneity, spatial and temporal distribution of events in the activity environment impose some restrictions on the synchronization over time of computational processes and events.

Solving the stated problem can be achieved by applying a calculation algorithm defined by operators of time logic based on events and probability $O(\tau) : \{Op_i(\tau), Op_2(\tau), \cdots Op_I(\tau)\}$ where: $O(\tau) \in \{\vee, \wedge, \cup, \cap\}$ - is the set of operators, $Op_i(\tau) = \{Ev_i[T], p_i(t)\}, i = \overline{(1, I)}$ - is the set of operands formed from the event $Ev_i[T]$ and the coefficient of decision-making influence

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$$p_i(t) = Ev_i[T] \Big/ (\frac{1/k + t^2}{\beta}), t = \overline{-\infty, +\infty}$$

where: $k \in [0;1]$- is the credibility coefficient of the event, t - is the time interval during which the coefficient of decisional influence is evaluated, β - is the coefficient of decisional stability.

Synchronization models of past-dependent events have been investigated $t = \overline{(-\infty, T)}$, for present $t = \overline{(-\infty, +\infty)}$ and future $t = \overline{(T, +\infty)}$.

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PARETO-NASH-STACKELBERG GAME AND CONTROL THEORY: STATE OF THINGS, TRENDS OF DEVELOPMENT, RELATION WITH GAMES DESIGN

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We present shortly the present state of things in the domain of Pareto-Nash-Stackelberg game and control theory [1]. Theory is treated as an integration, abstraction, and development of three distinct branches of classical game theory and the optimal control theory. The current state of things in Pareto-Nash-Stackelberg game and control theory is presented by referring mathematical models of various dynamic processes with concrete characteristics and parameters. Among different examples, we present analyses and investigation for the problem of linear discrete-time Pareto-Nash-Stackelberg control of decision processes that evolve as Pareto-Nash-Stackelberg games with constraints (a mixture of hierarchical and simultaneous games) under the influence of echoes and Ψ phenomena. We present mathematical models, solution notions, conditions for Pareto-Nash-Stackelberg control existence and method for Pareto-Nash-Stackelberg control computing. We expose, too, Wolfram Mathematica applications, demonstrations, benchmarks, and trend of theory development.

Finally, we analyze relations of the presented theory with the domain of game design.

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DECISION SUPPORT SYSTEM FOR MONITORING OF PATIENTS WITH DIABETES

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Recent research in medicine has shown that for a quality life in patients with diabetes mellitus, continuous monitoring is required, carried out by the patient and the specialist doctor [1,2]. Continuous blood glucose control requires enormous technical and technological resources. For real-time administration of blood glucose parameters - indispensable are the devices for data processing, their storage in the data base, effective analysis of values and prompt intervention in case of critical situations.

The researches carried out are oriented towards the development of a mixed support system in the conduct of the patient with diabetes mellitus. User side (patient) – calculation of physiological parameters, familiarity with new technology in the treatment of diabetes. Parting admin (endocrinologist) – monitoring and correction of errors occurred in the implementation of the diet calculated with the assessment of insulin requirements. The functionality of the system is based on the application of a Neural Network model. The learning process of the Neural Network takes place under the patient's home surveillance regime (remote monitoring of blood glucose parameters).

The monitoring and decision support process provides for the record of carbohydrate consumption, the fluctuation of blood glucose and the correlation of the amount of insulin administered in relation to blood sugar.

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V. DIDACTICS OF MATHEMATICS AND INFORMATICS

BAC-ul LA MATEMATICĂ ÎN REPUBLICA MOLDOVA: PROBLEME ŞI PERSPECTIVE

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Este important ca atât toţi actorii educaţionali, cât şi factorii de decizie din educaţie să conştientizeze că evaluarea influenţează enorm asupra procesului educaţional, inclusiv asupra predării şi învăţării. Cu atât mai mult asupra acestora influenţează examenele de absolvire a liceului. În fiecare an de studii fiecare elev al clasei a XII-a şi profesorii de matematică se întreabă: *Oare ce va fi la BAC anul acesta? Cum va fi structurat testul? Ce tipuri de itemi vor fi propuşi? Cum se vor evalua competenţele specifice, care trebuiau să fie formate? Cum se va ţine cont de Standardele de eficienţă a învăţării matematicii?*

Constatăm, cu mare regret că, deja mulţi ani, BAC-ul la matematică în Republica Moldova nu corespunde cerinţelor paradigmei de formare şi dezvoltare a competenţelor.

Analăzând testele propuse spre rezolvare în cadrul examenului de bacalaureat la matematică constatăm că nu se evaluează competenţele specifice, determinate de Curriculumul disciplinar la **Matematică** (nu curriculumul *la Algebră, la Geometrie* sau *la Analiză matematică!*), ci doar unele cunoştinţe din Algebră, Analiză matematică şi Geometrie. Chiar şi testul este divizat pe aceste trei componente/discipline?! Dar unde-i MATEMATICA?

Obiectivul major (pe care ar trebui să-l conştientizăm şi să-l realizăm în practică!!!) al examenului de absolvire a liceului la matematică este de a evalua la ce nivel sunt formate competenţele specifice matematicii determinate de curriculum şi competenţele-cheie determinate de Codul Educaţiei al Republicii Modova.

Ar trebui să învăţăm de la Franţa ce înseamnă bacalaureat la matematică (şi nu doar!) în bază de competenţe! Recomand profesorilor de matematică, echipei de elaborare a testelor de BAC, factorilor de decizie din Republica Moldova să analizeze şi să ţină cont de experienţa Franţei (ţării care prima în istorie a introdus bacalaureatul în practica educaţională!) privind evaluarea în bază de competenţe, inclusiv la matematică.

SISTEM SUPORT INTELIGENT APLICAT LA ORELE DE MATEMATICĂ DIN GIMNAZIU

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Această lucrare prezintă relevanţa inteligenţei artificiale în predarea matematicii prin utilizarea unui Sistem Suport Inteligent realizat în Wolfram Mathematica. Principalele componente ale SSI sunt următoarele:

1) baza de cunoştinţe SSI constând din:
   (a) setul de familii de sarcini specifice;
   (b) setul de modele generice ale familiilor de sarcini specifice;
2) baza de date SSI;
3) compozitorul sarcinilor personalizate pentru fiecare elev;
4) rezolvatorul sarcinilor matematice obţinute la etapa 3.

Partea inovatoare a acestei abordări constă în extinderea platformelor standard de e-learning, prin componenta inteligentă care facilitează sarcinile personalizate. Fiecare problemă specifică este generată şi personalizată automat de fiecare dată când studentul o accesează. Sarcinile iniţiale pot fi extinse pentru un număr nelimitat de variabile astfel încât fiecare elev dintr-o clasă, la fiecare acces, să primească un exerciţiu diferit, dar cu acelaşi grad de dificultate, iar metodologia folosită poate fi extinsă şi la celelalte unităţi de îvăţare din programa de matematică. În acest fel, se pot obţine rezultate practice remarcabile datorită evaluării obiective a fiecărui elev, eliminând riscul tentativelor de a copia.

APLICAŢIILE PROPRIETĂŢILOR FUNCŢIILOR ÎN REZOLVAREA ECUAŢIILOR

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Este cunoscută utilizarea funcţiilor strict monotone în rezolvarea ecuaţiilor şi inecuaţiilor [1-3]. În acelaşi context, vom prezenta aplicarea proprietăţilor de mărginire, paritate şi periodicitate ale funcţiilor elementare în rezolvarea unor ecuaţii.

O atenţie deosebită va fi acordată aplicaţiilor funcţiilor derivabile în rezolvarea ecuaţiilor, demonstrarea unor inegalităţi, determinarea numărului de soluţii ale unei ecuaţii, determinarea multiplicităţii rădăcinilor unui polinom etc.

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APLICAŢIILE PROGRAMĂRII DECLARATIVE ÎN REZOLVAREA TESTELOR DE INTELIGENŢĂ

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Prin cunoscuta programare declarativă înţelegem paradigma de programare în care programatorul defineşte *ceea ce trebuie îndeplinit* de program *fără a defini modul în care acesta trebuie să fie implementat.* Cu alte cuvinte, abordarea se concentrează *pe ceea ce trebuie atins în loc să instruiţi cum se poate realiza.* Deci, este diferit de un program imperativ care are setul de comandă pentru a rezolva un anumit set de probleme, descriind paşii necesari pentru a găsi soluţia. Instrumentele în programarea declarativă sunt furnizate programatorilor pentru a permite abstractizarea implementării şi pentru a ajuta la concentrarea problemei. Un program scris într-un limbaj logic constă dintr-un

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set de propoziţii, într-o formă logică, în care se exprimă fapte şi reguli despre problemă. Astfel, se descrie ce anume este o soluţie pentru problemă, nu modul în care se ajunge la ea. Soluţia este căutată în mulţimea de fapte, cu ajutorul setului de reguli. Această abordare de programare ajută la crearea codului mai inteligibil.

Un aspect aparte al programării declarative îl constituie simplificarea programării, reducând la minimum mutabilitatea. Structurile de date imuabile ajută la eliminarea erorilor greu de detectat şi sunt mai uşor de gestionat. În plus, se reduc efectele secundare ale stării, favorizând utilizarea unor constructe complexe, cum ar fi funcţii şi conducte de ordin superior şi descurajând variabilele.

În această ordine de idei, în raport vor fi prezentate probleme rezolvate cu ajutorul programării declarative, inclusiv popularul test de inteligenţă al lui Einstein. Albert Einstein a scris în secolul trecut un test de inteligenţă, care ar trebui rezolvat în cel mult 20-30 de minute. Geniul german punea problema într-un mod aparent simplu, dar cu un număr extraordinar de variabile. Însă, autorul la ceva timp după compunerea testului de logică a declarat, că numai 2% dintre cei care au încercat să-l rezolve au reuşit să obţină soluţia. Deci, mai în glumă mai în serios, acceptăm provocarea de a ne regăsi printre cei ce rezolvă cu succes testul de inteligenţă al lui Einstein.

METODE DE APLICARE A INTELIGENŢEI ARTIFICIALE LA DISCIPLINA ”TESTARE SOFTWARE AUTOMATIZATĂ”

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În lucrare sunt analizate metodele de aplicare a inteligenţei artificiale (IA) la disciplina Testare Software Automatizată (TSA). Este expus modul în care IA poate îmbunătăţi metodele existente de predare-învăţare-evaluare prin introducerea algoritmilor inteligenţi, modelelor de învăţare automată şi abordărilor bazate pe date.

Adoptarea tehnologiilor de IA a adus progrese semnificative în diverse domenii, inclusiv în disciplina TSA. Prin aplicarea metodei *Vizualizarea Conceptelor* în predare, cadrele didactice pot crea materiale de învăţare dinamice şi interactive care îmbunătăţesc implicarea, înţelegerea şi reţinerea conceptelor cheie pentru studenţi. Metoda *Căile de învăţare personalizate*, bazată pe analiza

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performanţei studenţilor, ajută la identificarea lacunelor de cunoştinţe şi oferirea resurselor, exerciţiilor adaptate la nevoile individuale ale studenţilor, oferind experienţe de învăţare personalizate şi adaptive. În contextul evaluării rezultatelor, metoda *Detectare a Plagiatului* poate asigura integritatea academică şi promova munca originală, prin compararea submisiilor de cod şi identificarea posibilelor cazuri de plagiat.

Aplicarea IA la disciplina TSA reprezintă o extindere a instrumentelor de predare-învăţare-evaluare, adăugându-le o componentă inteligentă de dezvoltare a conţinutului educaţional. Aceste avansuri tehnologice aduc beneficii atât pentru cadrele didactice, cât şi pentru studenţi, contribuind la dezvoltarea şi progresul în domeniul TSA.

ABORDĂRI ŞI STRATEGII DIDACTICE PENTRU PREDAREA CURSULUI BAZELE STATISTICII

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Procesul decizional în orice domeniu de activitate presupune luarea deciziilor informate, bazate pe date. Aceasta implică colectarea, analiza şi interpretarea datelor relevante. Instrumentele şi tehnicile de analiză şi interpretare a datelor sunt oferite de Statistică.

Toate programele de formare a specialiştilor în domeniul Ştiinţelor economice includ studierea Bazelor Statisticii. Pentru a asigura succesul studenţilor este nevoie din partea cadrelor didactice să aplice strategii creative şi efective de predare-învăţare-evaluare.

Pentru stimularea implicării active a studenţilor în studiul Bazelor Statisticii recomandăm integrarea conţinuturilor teoretice cu rezolvarea de exemple relevante atât pentru domeniul economic cât şi pentru grupul de studenţi (învăţarea interactivă şi practică).

Pentru dezvoltarea gândirii critice a studenţilor şi aplicarea cunoştinţelor în situaţii concrete propunem focusarea atenţiei pe interpretarea rezultatelor şi organizarea discuţiilor, iar pentru reducerea timpului de realizare a calculelor considerăm necesară utilizarea programelor software statistice. Schimbul de idei între studenţi, activitatea colaborativă şi aplicarea cunoştinţelor statistice în situaţii reale pot fi stimulate prin metoda de învăţare prin proiect.

Pentru identificarea dificultăţilor şi adaptarea instruirii la necesităţile studenţilor recomandăm monitorizarea continuă a progresului studenţilor, evaluarea continuă şi oferirea unui feedback constructiv.

METODOLOGIA ELABORĂRII PROBLEMELOR DE CONCURS

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Concursurile, olimpiadele de informatică (elevi, studenţi), care sunt destul de diverse prin forma sa, au devenit în prezent foarte populare. Ţinând cont de dezvoltarea rapidă a societăţii informaţionale, înaintarea cu paşi gigantici a inteligenţei artificiale în viaţa cotidiană, concursurile de informatică continuă să ruleze chiar şi după finalizarea oficială a acestora prin conţinutul lor. O etapă esenţială în organizarea competiţiilor de programare este crearea problemelor de concurs. De fapt, problemele de concurs reprezintă imaginea unei competiţii de programare şi ele determină viaţa de după concurs (sunt utilizate în calitate de instrumente pentru pregătirea de noi competiţii). Elaborarea problemelor de concurs pentru orice etapă (locală, raională, de sector, municipală, naţională, internaţională) este o sarcină destul de dificilă pentru autori, motiv pentru care necesită abordări metodice complexe, care ţin cont de: pregătirea teoretică şi practică a participanţilor; aspectul psihic şi situaţiile de stres; complexitatea concursului etc. Pentru elaborarea unei probleme de concurs autorul are nevoie de o pregătire foarte înaltă. Evidenţiem aici importanţa matematicii pentru crearea şi rezolvarea problemelor de concurs la informatică. O problemă de concurs are o structură formată din: fabulă, sarcină, date de intrare şi date de ieşire, restricţiile problemei, inclusiv timp şi memorie, exemple pentru claritate, imagini. De regulă, la baza creării unei probleme de concurs stă un set de noţiuni matematice sau se merge pe ideea utilizării unor structuri de date sofisticate sau – ambele. Formularea unei probleme de concurs trebuie să fie clară, fără a admite ambiguităţi. Evident, verificarea corectitudinii algoritmului creat este primordială.

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IMPACTUL INSTRUIRII PROBLEMATIZATE LA INTRODUCEREA UNUI NOU CONCEPT DE GEOMETRIE ÎN CLASA A VI-A

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Eli Passow afirma că la dezvoltarea oricărui concept, nu doar în matematică, se trece printr-o serie de etape, dintre care cea mai importantă este motivaţia [1]. La această etapă, se analizează necesitatea existenţei şi creării conceptului nou. De exemplu, elevii din clasele gimnaziale descoperă conceptul de *lungime a cercului şi* de *numărul* π, în clasa a VI-a. Ei se ciocnesc cu diferite neclarităţi şi cu foarte multe întrebări, care mai târziu pot duce la un dezinteres faţă de disciplina matematica. Pentru a evita acest lucru, o recomandare ar fi să introducem corect, interesant, interactiv şi adaptat vârstei conceptul de *lungime a cercului şi* de *numărul* π.

Elevii claselor a VI-a sunt foarte capabili, curioşi şi creativi. Pentru a li se menţine şi spori motivaţia în cadrul orelor de matematică recomandăm utilizarea metodei *problematizarea* - metodă interactivă, euristică şi modernă.

Problematizarea este modalitatea de a crea în mintea elevului o stare conflictuală (critică sau de nelinişte) intelectuală pozitivă, determinată de necesitatea cunoaşterii unui obiect, fenomen, proces sau rezolvării unei probleme teoretice sau practice pe cale logico-matematică şi (sau) experimentală [2].

Revenind la introducerea conceptului de *lungime a cercului şi* de *numărul* π, elevilor din clasa a VI-a li s-a propus următoarea situaţie problemă: "Perimetrul oricărui poligon uşor îl puteţi determina, dar cum ar fi cu perimetrul cercului? Există o formulă care ne-ar putea ajuta rapid să determinăm perimetrul cercului, ştiind unele date despre el?". Fiind împărţiţi în grupe de lucru, fiecare grup având pe masă câte o cutie, cu obiecte în formă de cerc, din diferite domenii: ştiinţă, tehnică, inginerie, arte şi matematică şi având la îndemână doar metrul de croitorie, rigla gradată, calculator de buzunar, un stilou şi un tabel pentru înregistrarea datelor, elevii au determinat formula de calcul pentru calculul perimetrului sau lungimii cercului.

În concluzie, putem afirma că adesea, câteva probleme aparent diferite, extrase dintr-o varietate de domenii din cadrul şi din afara matematicii, se dovedesc a fi strâns legate şi ne conduc la formularea unui concept general. Iar introducerea corectă a unui concept din domeniul matematicii şi utilizarea metodei corespunzătoare, problematizarea, conduc spre creşterea unei generaţii creative, cu o imaginaţie bogată şi soluţii organizaţionale. Problematizarea îi implică activ pe participanţi în gândirea creativă şi contribuie la dezvoltarea unor aptitudini, pentru că "intelectul se dezvoltă prin munca ce stimulează intelectul... Capacitatea minţii, excelenţa intelectuală reprezintă rezultatul" [3].

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PROMOVAREA E-TRANSFORMĂRII ÎNVĂŢĂMÂNTULUI PROFESIONAL TEHNIC PRIN COLABORARE

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Conform tendinţelor moderne globale şi Strategiilor naţionale de dezvoltare ”Moldova Europeană – 2030" şi "Educaţia – 2030”, realizarea cu succes a e-transformării Învăţământului Profesional Tehnic (ÎPT) presupune un accent mai mare pe colaborare, inovare şi promovarea mai hotărâtă a competenţei digitale prin diverse evenimente, gen mese rotunde, ateliere de lucru, conferinţe naţionale/internaţionale etc.

Întru realizarea acestui deziderat, de către autori, la data de 03.02.2023 a fost organizată o Masă rotundă pentru cadrele didactice şi managerii din ÎPT cu genericul ”Electronic Distance Learning (EDL) centru – un pas important spre dezvoltarea unui ecosistem educaţional inovativ prin e-Transformare sustenabilă şi durabilă a ÎPT”.

La eveniment au participat peste 100 de reprezentanţi din ÎPT, printre care Alexeev Tatiana, Managerul proiectului ”Tekwill în fiecare şcoală”, Mocanu Nicolae, Managerul de proiecte Cahul, Dimitriu Angela, manager de program ”eTwinning”, UTM, manageri şi cardre didactice din cadrul instituţiilor publice din ÎPT. În cadrul mesie rotunde au fost mai multe comunicări şi numeroase discuţii privind eficientizarea procesului de e-transformare a ÎPT. La finalul evenimentului au fost aplicate două sondaje de opinii: a) Privind utilizarea platformei educaţionale Moodle ca bază pentru EDL în ÎPT şi b) Aprecierea evenimentului propriu-zis.

Principalele concluzii se referă la: 1. *Necesitatea colaborării mai strânse Între instituţiile ÎPT, MEC;*

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2. *Formarea continuă a cadrelor didactice prin programe susţinute de MEC;*

3. *Coordonarea dezvoltării resurselor educaţionale digitale (RED) pentru ÎPT, inclusiv prin utilizarea potenţialului oferit de Tekwill, eTwinning, Comunitatea profesorilor din ÎPT pe Facebook (cu peste 1787 membri), Asociaţia Obştească "Educaţie pentru Dezvoltare" (AED) etc.*

S-a decis crearea/administrarea unui Registru unic al RED deschise, cu amplasarea pe Cloud sau în centre destinate, accesibile pentru toţi utilizatorii din ÎPT, şi care pot fi continuu îmbunătăţite în baza feedback-ului sistematic şi analizei retrospective speciale.

ON THE ABILITY OF ENGINEERING STUDENTS TO GET CORRECT ANSWERS IN MATHEMATICS EXAMINATIONS

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The results of engineering students in mathematics examinations where marks are awarded only for correct answers are compared with those where partial marks are awarded. Data from two courses are analysed and conclusions are drawn.

EXPLORAREA FIŞIERELOR WINDOWS BMP

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Formatul grafic Windows BMP este descris detaliat în mai multe surse, precum şi în suportul de curs [1]. Pentru a uşura studierea acestui format de către studenţii care frecventează cursul de Grafică 2D pe calculator, autorii au elaborat o aplicaţie sub formă de casetă de dialog, având interfaţă tipică ferestrelor sistemului de operare Windows. Aplicaţia a fost realizată în limbajul C#, utilizând biblioteca .Net Framework, şi numită ExploringBMP. Interfaţa ei este compusă din butoane de comandă, etichete statice, casete de editare, casete de validare şi trei panouri. Partea principală şi cea mai mare a suprafeţei de lucru aparţine panoului umplut cu casete de editare mici, care asigură vizualizarea,

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crearea şi modificarea codului hexazecimal al imaginii în formatul BMP. Toate acţiunile disponibile sunt precizate suplimentar prin explicaţii flotante.

Aplicaţia oferă posibilităţile de a încărca şi a vizualiza codul hexazecimal al conţinutului fişierului BMP indicat; de a ascunde/afişa: antetul fişierului, antetul imaginii codificate în acest fişier, harta de culori a imaginii codificate în acest fişier (în cazul adâncimii 1, 4 sau 8 biţi la un pixel), codul imaginii codificate în acest fişier. Implicit după încărcarea fişierului toate antetele, harta de culori a imaginii (dacă ea este prezentă), codul imaginii sunt afişate. În cazul adâncimilor 8 sau 24 biţi la un pixel, codul imaginii codificate poate fi colorat cu culorile respective sau nu. În cazul când harta de culori este prezentă, codul ei tot poate fi colorat.

Au fost realizate şi câteva funcţii suplimentare, cum ar fi ascunderea/afişarea spaţiului neutilizat, afişarea într-o casetă de dialog nemodală a tabelului ASCII, inserarea porţiunilor de cod şi de text începând de la poziţia indicată.

Codul modificat al imaginii poate fi salvat (Save) în fişier sub acelaşi nume cu care a fost încărcat, sau sub un nume nou (Save As).

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STUDIEM METODE DE PROGRAMARE PRIN ALGORITMI DE SORTARE

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Nu se ştie precis cine, dar totuşi cineva a exprimat primul că ”Totul se cunoaşte în comparaţie”. În [2] autorii au demonstrat aplicarea acestei expresii la demonstrarea a 5 metode de implementare a algoritmilor, care au devenit demult clasice. Este vorba de ”Metoda algoritmului lacom (engl. Greedy algorithm)”, ”Principiul *Împarte şi stăpâneşte* (lat. *Divide et impera*)”, ”*Metoda de căutare cu revenire* (eng. *Backtracking*)”, ”*Metoda programării dinamice* (eng. *Dynamic Programming*)” şi ”*Metoda cu utilizarea abordării euristice* (eng. *Heuristic Programming*)”. Pentru demonstrarea fiecărei din aceste metode sunt bine cunoscute exemple clasice de probleme respective. Autorii propun începătorilor studierea acestor metode prin rezolvarea uneia şi aceleiaşi probleme clasice şi foarte importantă, şi anume a problemei de sortare internă a vectorilor [1].

[42] *Speaking author*: A. Pereteatcu

Astfel au fost demonstrate prin algoritmi de sortare bine cunoscuţi: "*Metoda algoritmului lacom*" prin algoritmul trivial "*Selecţie simplă*"; "Principiul *Împarte şi stăpâneşte*" în varianta "moale" prin sortare prin interclasare; "Principiul *Împarte şi stăpâneşte*" în varianta "strictă" prin sortare rapidă. Pentru a demonstra "*Căutarea cu revenire*", "*Programarea dinamică*" şi "*Programarea euristică*" au fost elaborate clase dotate cu algoritmi de sortare respectivi. Pentru fiecare algoritm de sortare au fost estimate complexităţile teoretice atât temporale cât şi cele spaţiale. Complexităţile teoretice au fost comparate cu complexităţile practice, obţinute prin exemple. Au fost propuse multe exerciţii, rezolvarea cărora va fi de folos pentru cei care vor studia materialul la tema dată.

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UNELE PRINCIPII ŞI RECOMANDĂRI ÎN RAPORT CU PREGĂTIREA MATERIALELOR DIDACTICE AUDIO-VIZUALE PENTRU UNITATEA DE CURS TEHNOLOGII INFORMAŢIONALE ŞI COMUNICAŢIONALE

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În acest articol sunt descrise unele principii şi recomandări în raport cu pregătirea materialelor didactice audio-vizuale pentru unitatea de curs *Tehnologii Informaţionale şi Comunicaţionale* (TIC). La etapa actuală una dintre principalele probleme care foarte mult îi frământă pe profesori, constă în găsirea celor mai bune modalităţi de predare a materiei, în determinarea eficacităţii metodelor aplicate în predarea cursurilor universitare. În opinia cercetătorilor sunt reliefate trei stiluri prin care studenţii pot asimila mult mai rapid materia predată: (1) Stilul kinestezic de învăţare; (2) Stilul auditiv de învăţare; (3) Stilul vizual de învăţare.

Din cele enumerate, *stilul vizual de învăţare* se remarcă printr-o listă complexă de modalităţi practice prin care aceştia pot asimila materia. Studenţii care preferă stilul vizual de învăţare, de cele mai multe ori sunt foarte creativi şi au propria lor percepţie asupra materiei predate.

[43] *Speaking author*: L. Popov

În cadrul lecţiilor, la unitatea de învăţare TIC, care sunt preponderent practice este binevenită, dar în multe cazuri chiar necesară, folosirea materialelor audio-vizuale pentru a facilita studenţilor cunoaşterea programelor studiate, memorizarea paşilor de implementare a unor procesări complexe a informaţiei, înţelegerea proceselor informatice şi, în rezultat, formarea competenţelor digitale corespunzătoare.

În articol se expun unele cerinţe/principii referitoare la pregătirea materialelor didactice audio-vizuale (restricţii referitoare la lungimea secvenţelor audio-video, conţinutul secvenţelor, în ce măsura trebuie de însoţit video cu coloana sonoră etc.). De asemenea, se scot în relief avantajele/dezavantajele utilizării aşa unor materiale, inclusiv condiţii care oferă această posibilitate (de exemplu, prezenţa calculatoarelor şi a sistemului multimedia (căşti, boxe etc.)) şi în cele din urmă prezentarea unor recomandări pentru pregătirea materialelor didactice audio-vizuale atât pentru unitatea de curs TIC, cât şi pentru alte unităţi de curs fie universitare sau preuniversitare.

SOME METHODICAL ASPECTS TO CALCULUS OF LAPLACE TRANSFORM WITH MATHEMATICAL PACKAGES

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The Laplace transform has many important applications in mathematics, physics, optics, electrical engineering, automation, signal processing and probability theory. It is also a tool for solving differential equations.

In Higher Mathematics one of modules is dedicated to the Laplace transform. If the function $f(t)$ is an *original function*, then the Laplace transform is defined by [1] $\mathfrak{L}f(s) = \int\limits_0^\infty f(t)e^{-st}dt$, where s is a complex frequency domain parameter $s = a + bi$ with real numbers a and b.

In practice, it is not simple to calculate a Laplace transform for a function $f(t)$, it is necessary to use some proprieties of Laplace transform for this [2] but this one offers specific facilities which helps solve and interpret various problems.

The main objective of this paper is to finding the Laplace transform of some functions using special programmes Maple [3], Wolfram Mathematica [4]. Digital applications are useful in the teaching process, in determining of the Laplace transform that can be used to solve various concrete problems.

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INTELEGENŢA ARTIFICIALĂ, FACILITĂŢI ŞI PROVOCĂRI PENTRU DIDACTICA MATEMATICĂ

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Dacă pui întrebare unui Sistem de Inteligenţă Artificială (Ex. Chat GPT) ”Ce este intelegenţa artificială?” el îţi va răspunde: ”*Inteligenţa artificială (IA) este un domeniu al informaticii care se ocupă cu crearea de sisteme şi maşini care pot imita şi simula comportamentul uman inteligent. Scopul principal al inteligenţei artificiale este de a dezvolta sisteme capabile să înveţe, să rezolve probleme, să ia decizii şi să îndeplinească sarcini într-un mod similar cu fiinţele umane. Acestea pot fi proiectate să efectueze activităţi precum recunoaşterea vocală şi facială, traducerea automată, analiza şi clasificarea datelor, planificarea şi optimizarea, jocuri de strategie, asistenţă virtuală şi multe altele*”.

Deci, acestea sunt facilităţi, puse la dispoziţia cadrelor didactice, elevilor, studenţilor şi celor care doresc să studieze, care trebuie implementate şi valorificate la toate nivelele de învăţare.

Dar, apar şi provocări, iar eliminarea efectelor produse de acestea, pe viitor, probabil va fi introdusă ca o materie de studiu, pentru cadrele didactice în devenire.

Care ar fi acele provocări, care sunt identificate de un cadru didactic, misiunea acestuia fiind nu doar de a învăţa copiii unele materii ci şi ai dezvolta multilateral personalitatea, identificate chiar şi la ziua de azi?

Problemele de matematică reflectă situaţii de viaţă, iar rezolvarea lor dezvoltă la elevi un simţ al realităţii de tip matematic şi creează premise ale succesului rezolvării unor probleme din viaţa cotidiană. Rezolvarea problemelor de matematică în procesul formării personalităţii elevului este extrem de importantă. În cadrul acestor acţiuni se dezvoltă procesele cognitive, volitive

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şi motivaţional-afective, se favorizează formarea competenţelor de muncă intelectuală, se stimulează creativitatea şi flexibilitatea gândirii, se dezvoltă capacităţile de anticipare şi perspicacitate, se educă spiritul de iniţiativă, se fortifică încrederea în forţele proprii etc.

Şi atunci, utilizarea sistemelor de IA, care au capacitatea de a rezolva unele dintre cele mai dificile probleme, ar reduce la minim efortul celui care le folosesc pe larg. Aici şi apare una dintre primele provocări, iar cadrele didactice vor trebui să facă faţă acestora, zilnic.

În concluzie. Este îmbucurător faptul că societate progresează, apar noi tehnologii, capabile să rezolve situaţii şi probleme foarte dificile, astfel făcând viaţa noastră mai uşoară şi mai interesantă. Misiunea cadrului didactic însă rămâne aceeaşi, de a formula sarcini şi de a crea situaţii pentru copii, ca aceştia să se dezvolte ca personalităţi intelegente, folosind toate facilităţile care le pun la dispoziţia lor societatea modernă.

UTILIZAREA METODEI PROBLEMELOR TRANSVERSALE LA PREDAREA MATEMATICII ECONOMICE

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În procesul de instruire matematică, rezultatele profesorului depind, într-o mare măsură, de capacitatea studenţilor de a însuşi materia predată. Astfel, cadrul didactic este într-un proces continuu de căutare şi dezvoltare a noilor metode de predare cu implementarea noilor tehnologii.

În acest articol, succint, va fi descrisă una dintre metodele care permit predarea şi însuşirea cu succes a disciplinelor matematice, în cadrul altor specialităţi universitare. Este vorba despre *Metoda Problemelor Transversale (MPT)*, propusă de Vilenkin N.Ya [1].

Metoda Problemelor Transversale, cunoscută şi sub denumirea de *Metoda lui Vilenkin*, este o tehnică matematică utilizată în rezolvarea ecuaţiilor diferenţiale parţiale eliptice într-un domeniu închis. Aceasta se bazează pe ideea de a transforma problema de frontieră dată într-o problemă transversală, care este mai uşor de rezolvat. Este o tehnică puternică şi flexibilă, care a fost extinsă şi generalizată în diverse direcţii.

Este important de menţionat că MPT este un subiect avansat în matematică şi necesită cunoştinţe solide ale studenţilor pentru a fi înţeleasă şi aplicată corect.

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Însuşirea acestei metode ar stimula gândirea lor critică, ar dezvolta abilităţile de analiză şi sinteză, cât şi a celor de aplicare a conceptelor teoretice la rezolvarea problemelor practice reale din diferite domenii.

Cum a fost menţionat, utilizarea MPT în predarea matematicilor avansate la alte specialităţi, precum ar fi cele din cadrul ştiinţelor economice, presupune formularea unei singure probleme complexe reale, din domeniul de specialitate, rezolvarea căreia ar necesita aplicarea cunoştinţelor dobândite în cadrul diferitor disciplini ale matematicii (algebră, geometrie analitică, etc.) [2].

Ca exemplu de problemă transversală, propusă la disciplina Matematica Economică, este *problema optimizării planului de producţie al unei întreprinderi.* Luând în calcul restricţiile resurselor producerii, se cere: 1) elaborarea unui model matematic al problemei; 2) rezolvarea problemei de programare liniară, utilizând metoda grafică pentru a determina planul de producţie optim; 3) rezolvarea aceleiaşi probleme de programare liniară, utilizând metoda simplex pentru a identifica soluţia optimă; 4) obţinerea soluţiei generale pentru problema de programare liniară iniţială.

Acest exemplu permite studenţilor să înţeleagă cum pot fi aplicate metodele matematice într-o situaţie economică reală şi îi ajută să dezvoltă abilităţi de utilizare a cunoştinţelor matematice în scenarii practice.

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THE SOLUTIONS OF SOME DIOPHANTINE EQUATIONS

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In the theory of Diophantine equations, is well known the equation $a^x + b^y = z^2$. The literature contains a very large number of articles on such equations [1–6].

In this paper, we solve the equations:

$$3^x + b^y = z^2, b \in \{40, 360, 3240, 29160\},$$

where x, y, z are non-negative integer numbers.

Theorem 1. *The Diophantine equation* $32^x + 40^y = z^2$ *has exactly five integer non-negative solutions* $(x, y, z) \in$*{(1,0,2),(2,1,7), (2,3,253), (4,1,11), (4,2,41)}.*

Theorem 2. *The Diophantine equation* $3^x + 360^y = z^2$ *has exactly five integer non-negative solutions* $(x, y, z) \in$ *{(1,0,2),(4,1,21), (6,1,33), (8,2,369), (8,3,6831)}.*

Theorem 3. *The Diophantine equation* $3^x + 3240^y = z^2$*has exactly six-integer non-negative solutions* (x, y, z) $\in$*{(1,0,2),(2,1,57), (6,1,63), (8,1,99), (12,2,3321), (14,3,184437)}.*

Theorem 4. *The Diophantine equation* $3^x + 21960^y = z^2$ *has exactly six integer non-negative solutions* $(x, y, z) \in$ *{(1,0,2),(4,1,171), (8,1,189), (10,1,297), (16,2,29889), (20,3,4979799)}.*

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DESIGN-UL DOMENIILOR DE DEZVOLTARE A ELEVILOR CAPABILI DE PERFORMANŢE ÎNALTE LA MATEMATICĂ

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Procesul de dezvoltare a competenţelor copiilor dotaţi la matematică este complex şi necesită a fi susţinut din exterior prin oferirea de resurse. Se consideră că studiile viitoare în domeniul excelenţei trebuie să mărească puterea statistică, iar studiile neuro-imagistice să se bazeze pe date comportamentale

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de susţinere atunci când interpretează constatările. Studiile ar trebui să investigheze factorii care s-au dovedit a corela cu talentul la matematică într-o manieră mai specifică şi să determine exact modul în care factorii individuali pot contribui la capacitatea matematică a dotaţilor.

Abordările metodologice de proiectare a lucrului cu copiii dotaţi la matematică sunt: abordarea sistemică, centrarea pe personalitate, abordarea individului ca poli-subiect, abordarea culturologică, individualizată, orientată spre dezvoltarea creativităţii şi altele [1]. Totalitatea acestora determină proiectarea unui sistem integru care include identificarea copiilor capabili de performanţe înalte, formarea şi dezvoltarea capacităţilor lor. În lucrarea [2] este propus Modelul pedagogic de dezvoltare a competenţelor copiilor dotaţi la matematică care vizează armonizarea dezvoltării competenţelor copiilor dotaţi la matematică bazat pe concepţia rezultantei şi crearea unui mediu de dezvoltare axat pe trei dimensiuni: *dezvoltarea intelectuală, dezvoltarea personală, dezvoltarea abilităţilor comunicaţionale.* Aceste trei dimensiuni interacţionează şi se suprapun, iar acţiunile în zonele de interferenţă presupun luarea în considerare a particularităţilor individuale ale discipolilor şi canalizarea eforturilor cadrelor didactice.

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APLICAREA PIVOT TABLE EXCEL ÎN SOLUŢIONAREA PROBLEMELOR PRACTICE

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În această lucrare este scoasă în relief aplicarea Pivot Table Excel în soluţionarea problemelor practice. A vorbi despre Pivot Tabel înseamnă că s-a ajuns la un nivel intermediar avansat în ceea ce priveşte utilizarea aplicaţiei Excel, practic în cazul în care se doreşte o analiză mai rapidă a informaţiei, printr-un alt mod de calcul a datelor. Pentru a lucra cu datele în Tabele Pivot este nevoie de a respecta câteva reguli importante şi anume: trebuie să existe o bază de date (BD) bine structurată cu date corect definite; formatări, corect aplicate

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asupra datelor; conţinut relevant în BD; să nu existe rânduri sau coloane goale în interiorul BD; să nu existe rânduri cu valori calculate etc.

Tabelul Pivot în Excel are rol de raport, este un instrument puternic de pregătire a listelor în vederea analizei sau tipăririi datelor. Anume datorită acestui tabel se sintetizează şi se prezintă într-un mod organizat şi foarte rapid informaţia de care este nevoie la un moment dat din BD. Tabelul Pivot prezintă un instrument deosebit de flexibil al mediului Excel, există numeroase posibilităţi de modificare sau restructurare precum şi de mărire a clarităţii şi interpretabilităţii acestuia, precum şi actualizarea datelor care, la rândul său, nu se realizează dinamic. Datele pe care le conţine Tabelul Pivot, pot fi reprezentate grafic. Un moment important în acest scop constă în faptul că există posibilitatea de a crea rapoarte finale atât dintr-un tabel, cât şi din mai multe tabele în cazul în care există legătura între acestea. Aplicând Tabelele Pivot în practică la soluţionarea unor probleme, avem de câştigat în timp şi de a depune foarte puţin efort. Sunt scoase în evidenţă avantajele şi dezavantajele acestora.

CONCEPTE DE APLICARE A MACRODEFINIŢIILOR VBA EXCEL ÎN PROBLEME DE CLASIFICARE

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Programarea în Excel se realizează prin limbajul de programare Visual Basic for Application, care este un subset de limbaj de programare Visual Basic puternic şi este iniţial încorporat în majoritatea aplicaţiilor Office [1].

Comenzile Macro reprezintă o unealtă Excel ce este utilizată atunci când acţiunile trebuie repetate asupra unor celule sau unor foi de calcul diferite. Aceste comenzi permit înregistrarea şi apoi repetarea acţiunilor [2]. La clasificarea diverselor date ale elevilor, studenţilor (angajaţilor unor companii, listei medicamentelor etc.) deseori apare necesitatea de partajare a informaţiei după diverse criterii.

În lucrare sunt cercetate posibilităţile de aplicare a macro-definiţiilor în Microsoft Excel şi programarea acestora pentru clasificarea datelor elevilor, studenţilor şi gestiunea listei medicamentelor.

Sunt descrise modalităţile de programare în Visual Basic for Application, de aplicarea a VBA în Microsoft Excel, este cercetat riscul de securitate a informaţiei, aplicând macro-definiţiile.

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Este descris modul de programare a macro-definiţiilor pentru gestiunea datelor personale ale elevilor/studenţilor. Sunt înscrise în următoarele coloane într-o foaie de calcul Excel: numărul de rând sau id; numele şi prenume elevului; clasa/grupa; adresa domiciliului, care include municipiul sau raionul şi satul; adresa email şi telefonul. Sunt definite macro-definiţii pentru selectarea în foi de calcul separate a elevilor pentru fiecare clasă, înscrierea în foi separate a informaţiei despre elevi/studenţi pe localităţi în foi aparte, precum şi înscrierea în foi aparte a informaţiei despre elevi/studenţi din diferite clase/ grupe în foi separate.

De asemenea, sunt definite macro-definiţii pentru gestiunea listei medicamentelor. Se cunoaşte o listă a medicamentelor dintr-o farmacie, înscrise în următoarele coloane într-o foaie de calcul Excel: numărul de rând sau id; denumirea medicamentului; forma de împachetare: comprimate, sirop, picături, spray, fiole, drajeuri, capsule etc.; compania şi ţara producătoare; categoria (diagnoza generală); denumirea internaţională, greutatea, termenul de garanţie şi preţul medicamentului. Sunt definite macro-definiţii pentru clasificarea medicamentelor pentru fiecare diagnoză în parte, formatarea listei medicamentelor şi ştergerea foilor de calcul create.

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Index

Contents